Fundamentals of Computer Systems

Thinking Digitally

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Columbia University

Fall 2013

The Subject of this Class

0

The Subjects of this Class

0 1

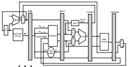
Engineering Works Because of Abstraction

Application Software

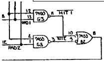
Operating Systems

```
;; voice 1 wave select
ld      a,(#CH1_W_NUM)
and     a
ld      a,(#CH1_W_SEL)
jr      nz,#00b4
ld      a,(#CH1_E_TABLE0)
```

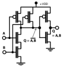
Architecture



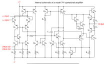
Micro-Architecture



Logic

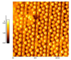


Digital Circuits



Analog Circuits

Devices

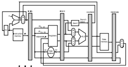
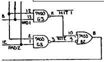
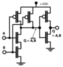
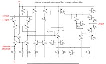
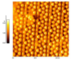


Physics

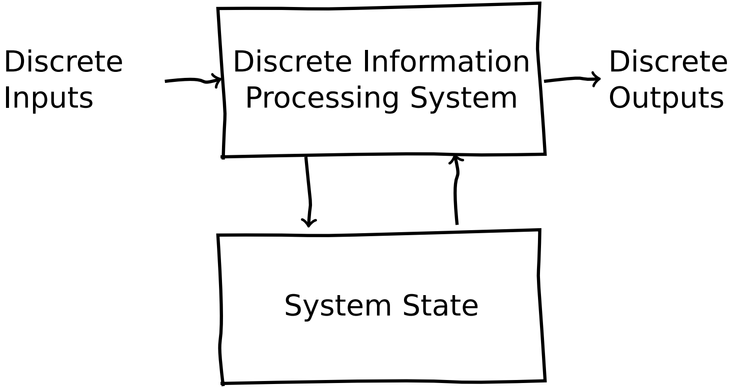
Engineering Works Because of Abstraction

	Application Software	COMS 3157, 4156, et al.
	Operating Systems	COMS W4118

`;; voice 1 wave select`
`ld a,(#CH1_W_NUM)`
`and a`
`ld a,(#CH1_W_SEL)`
`jr nz,#00b4`
`ld a,(#CH1_E_TABLE0)`
	Architecture	Second Half of 3827

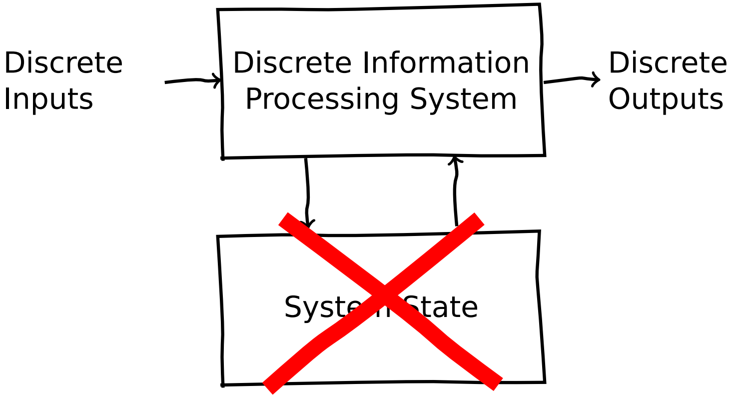
	Micro-Architecture	Second Half of 3827
	Logic	First Half of 3827
	Digital Circuits	First Half of 3827
	Analog Circuits	ELEN 3331
	Devices	ELEN 3106
	Physics	ELEN 3106 et al.

Simple information processing system



First half of the course

Simple information processing system



First quarter of the course

Administrative Items

Mailing list: csee3827-staff@lists.cs.columbia.edu

`http://www.cs.columbia.edu/~martha/courses/3827/au13/`

Prof. Martha A. Kim

martha@cs.columbia.edu

469 Computer Science Building

Lectures 10:10–11:25 AM Tue, Thur
209 Havemeyer
Sep 3–Dec 5
Holidays: Nov 5 (Election Day), Nov 28 (Thanksgiving)

Assignments and Grading

Weight	What	When
40%	Six homeworks	See Webpage
30%	Midterm exam	(Tentatively) October 15th
30%	Final exam	During Finals Week (Dec 13–20)

Homework is due at the beginning of lecture.

We will drop the lowest of your six homework scores;

$$\text{you can } \left\{ \begin{array}{c} \text{skip} \\ \text{omit} \\ \text{forget} \\ \text{ignore} \\ \text{blow off} \\ \text{screw up} \\ \text{feed to dog} \\ \text{flake out on} \\ \text{sleep through} \end{array} \right\} \text{ one with no penalty.}$$

Rules and Regulations

You may collaborate with classmates on homework.

Each assignment turned in must be unique; work must ultimately be your own.

List your collaborators on your homework.

Do not cheat.

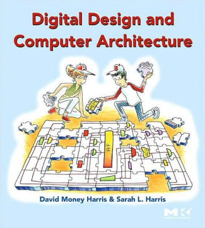
Tests will be closed-book with a one-page “cheat sheet” of your own devising.

The Text(s): Alternative #1

No required text. There are two recommended alternatives.

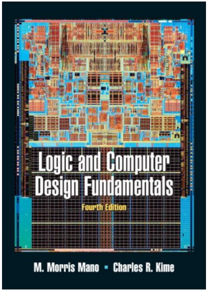
- David Harris and Sarah Harris. *Digital Design and Computer Architecture.*

  Almost precisely right for the scope of this class: digital logic and computer architecture.

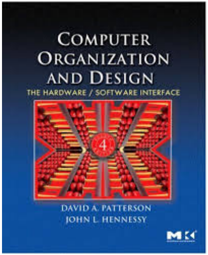
  

The Text(s): Alternative #2

- M. Morris Mano and Charles Kime. *Logic and Computer Design Fundamentals, 4th ed.*



- Computer Organization and Design, The Hardware/Software Interface, 4th ed. *David A. Patterson and John L. Hennessy*



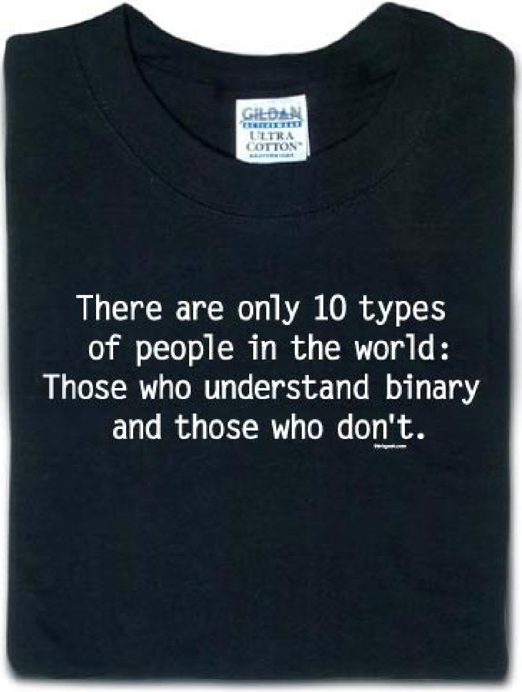
Registration and Wait List

This class is currently full.

If we can secure a larger lecture hall, we will increase cap.

The course has a waitlist:
`https://docs.google.com/forms/d/1GygsbDZvP64hAi0nNWYqBqeXIbS70CxMgROe5-5Y3q8/viewform`

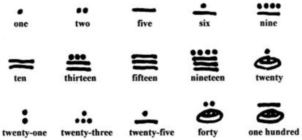
It is offered every semester.



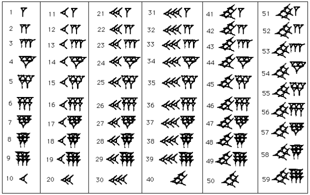
thinkgeek.com

Which Numbering System Should We Use? Some Older Choices:

Roman: I II III IV V VI VII VIII IX X



Mayan: base 20, Shell = 0



Babylonian: base 60

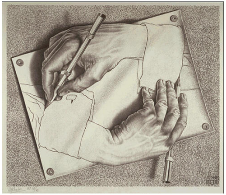
The Decimal Positional Numbering System

Ten figures: 0 1 2 3 4 5 6 7 8 9

$7 \times 10^2 + 3 \times 10^1 + 0 \times 10^0 = 730_{10}$

$9 \times 10^2 + 9 \times 10^1 + 0 \times 10^0 = 990_{10}$

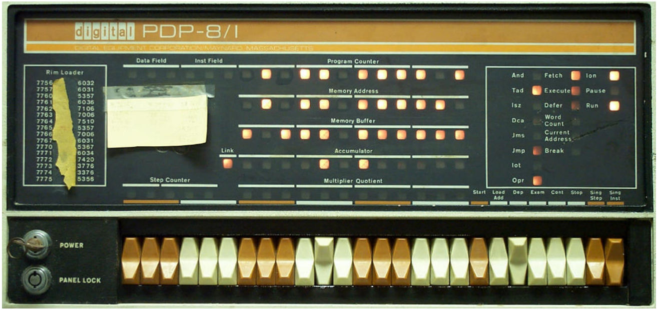
Why base ten?



Hexadecimal, Decimal, Octal, and Binary

Hex	**Dec**	**Oct**	**Bin**
0	0	0	0
1	1	1	1
2	2	2	10
3	3	3	11
4	4	4	100
5	5	5	101
6	6	6	110
7	7	7	111
8	8	10	1000
9	9	11	1001
A	10	12	1010
B	11	13	1011
C	12	14	1100
D	13	15	1101
E	14	16	1110
F	15	17	1111

Binary and Octal



DEC PDP-8/I, c. 1968

Oct	Bin
0	0
1	1
2	10
3	11
4	100
5	101
6	110
7	111

$$\begin{aligned} \text{PC} &= 0\times 2^{11}+1\times 2^{10}+0\times 2^{9}+1\times 2^{8}+1\times 2^{7}+0\times 2^{6}+ \\ &\quad 1\times 2^{5}+1\times 2^{4}+1\times 2^{3}+1\times 2^{2}+0\times 2^{1}+1\times 2^{0} \\ &= 2\times 8^{3}+6\times 8^{2}+7\times 8^{1}+5\times 8^{0} \\ &= 1469_{10} \end{aligned}$$

Hexadecimal Numbers

Base 16: 0 1 2 3 4 5 6 7 8 9 A B C D E F

Instead of groups of 3 bits (octal), Hex uses groups of 4.

$$
\begin{aligned}
\mathrm{CAFEF00D}_{16} &= 12 \times 16^7 + 10 \times 16^6 + 15 \times 16^5 + 14 \times 16^4 + \\
&\quad 15 \times 16^3 + 0 \times 16^2 + 0 \times 16^1 + 13 \times 16^0 \\
&= 3,405,705,229_{10}
\end{aligned}
$$

| C | A | F | E | F | 0 | 0 | D | Hex
11001010111111101111000000001101 Binary
| 3 | 1 | 2 | 7 | 7 | 5 | 7 | 0 | 0 | 1 | 5 | Octal

Computers Rarely Manipulate True Numbers

Infinite memory still very expensive

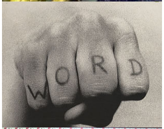
Finite-precision numbers typical

32-bit processor: naturally manipulates 32-bit numbers

64-bit processor: naturally manipulates 64-bit numbers

How many different numbers can you represent with 5 binary / octal / decimal / hexadecimal digits?

Jargon

	Bit	Binary digit: 0 or 1
	Byte	Eight bits
	Word	Natural number of bits for the processor, e.g., 16, 32, 64
	LSB	Least Significant Bit (“rightmost”)
	MSB	Most Significant Bit (“leftmost”)

Decimal Addition Algorithm

$$\begin{array}{r} 434 \\ +628 \\ \hline \end{array}$$

$$4 + 8 \quad = \quad 12$$

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18
10	10	11	12	13	14	15	16	17	18	19

Decimal Addition Algorithm

$$\begin{array}{r} \color{red}{1} \\ 434 \\ +628 \\ \hline \color{blue}{2} \end{array}$$

$$4 + 8 = \color{red}{1}\color{blue}{2}$$
$$1 + 3 + 2 = \color{blue}{6}$$

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18
10	10	11	12	13	14	15	16	17	18	19

Decimal Addition Algorithm

$$\begin{array}{r} 1 \\ 434 \\ +628 \\ \hline 62 \end{array}$$

$$\begin{aligned} 4+8 &= 12 \\ 1+3+2 &= 6 \\ 4+6 &= 10 \end{aligned}$$

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18
10	10	11	12	13	14	15	16	17	18	19

Decimal Addition Algorithm

$$\begin{array}{r} \color{red}{1\ \ 1\ \ } \\ 434 \\ +628 \\ \hline \color{blue}{062} \end{array}$$

$$\begin{aligned} 4+8 &= \color{red}{1}\color{blue}{2} \\ 1+3+2 &= \color{blue}{6} \\ 4+6 &= \color{red}{1}\color{red}{0} \end{aligned}$$

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18
10	10	11	12	13	14	15	16	17	18	19

Decimal Addition Algorithm

$$
\begin{array}{r}
\color{red}{1\;\;1} \\
434 \\
+628 \\
\hline
\color{blue}{1062}
\end{array}
$$

$$
\begin{aligned}
4+8 &= \color{red}{1}\color{blue}{2} \\
1+3+2 &= \color{blue}{6} \\
4+6 &= \color{red}{10}
\end{aligned}
$$

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18
10	10	11	12	13	14	15	16	17	18	19

Binary Addition Algorithm

$$\begin{array}{r} 10011 \\ +11001 \\ \hline \end{array}$$

$$1 + 1 \quad = \quad 10$$

+	0	1
0	00	01
1	01	10
10	10	11

Binary Addition Algorithm

$$\begin{array}{r} \color{red}{1} \\ 10011 \\ +11001 \\ \hline \color{blue}{0} \end{array}$$

$$\begin{aligned} 1+1 &= \color{red}{1}\color{blue}{0} \\ 1+1+0 &= \color{red}{1}\color{blue}{0} \end{aligned}$$

+	0	1
0	00	01
1	01	10
10	10	11

Binary Addition Algorithm

$$\begin{array}{r} \color{red}{11} \\ 10011 \\ +11001 \\ \hline \color{blue}{00} \end{array}$$

+	0	1
0	00	01
1	01	10
10	10	11

$$1+1 = \color{red}{1}\color{blue}{0}$$
$$1+1+0 = \color{red}{1}\color{blue}{0}$$
$$1+0+0 = \color{red}{0}\color{blue}{1}$$

Binary Addition Algorithm

$$
\begin{array}{r}
011 \\
10011 \\
+11001 \\
\hline
100
\end{array}
$$

$$
\begin{aligned}
1 + 1 &= 10 \\
1 + 1 + 0 &= 10 \\
1 + 0 + 0 &= 01 \\
0 + 0 + 1 &= 01
\end{aligned}
$$

+	0	1
0	00	01
1	01	10
10	10	11

Binary Addition Algorithm

$$
\begin{array}{r}
0011 \\
10011 \\
+11001 \\
\hline
1100
\end{array}
$$

$$
\begin{aligned}
1 + 1 &= 10 \\
1 + 1 + 0 &= 10 \\
1 + 0 + 0 &= 01 \\
0 + 0 + 1 &= 01 \\
0 + 1 + 1 &= 10
\end{aligned}
$$

+	0	1
0	00	01
1	01	10
10	10	11

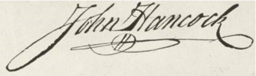
Binary Addition Algorithm

$$
\begin{array}{r}
\color{red}{10011} \\
10011 \\
+11001 \\
\hline
\color{blue}{101100}
\end{array}
$$

+	0	1
0	00	01
1	01	10
10	10	11

$$
\begin{aligned}
1+1 &= \color{red}{1}\color{blue}{0} \\
1+1+0 &= \color{red}{1}\color{blue}{0} \\
1+0+0 &= \color{red}{0}\color{blue}{1} \\
0+0+1 &= \color{red}{0}\color{blue}{1} \\
0+1+1 &= \color{red}{1}\color{blue}{0}
\end{aligned}
$$

Signed Numbers: Dealing with Negativity



How should both positive and negative numbers be represented?

Signed Magnitude Numbers

You are most familiar with this:
negative numbers have a leading $-$

In binary, a leading 1 means negative:

$0000_2 = 0$

$0010_2 = 2$

$1010_2 = -2$

$1111_2 = -7$

$1000_2 = -0?$

Can be made to work, but addition is annoying:

If the signs match, add the magnitudes and use the same sign.

If the signs differ, subtract the smaller number from the larger; return the sign of the larger.

One's Complement Numbers

Like Signed Magnitude, a leading 1 indicates a negative One's Complement number.

To negate a number, complement (flip) each bit.

$0000_2 = 0$

$0010_2 = 2$

$1101_2 = -2$

$1000_2 = -7$

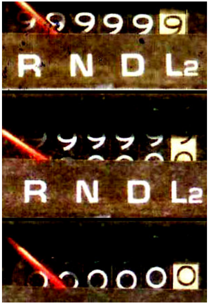
$1111_2 = -0?$

Addition is nicer: just add the one's complement numbers as if they were normal binary.

Really annoying having a -0: two numbers are equal if their bits are the same or if one is 0 and the other is -0.


NOT ALL
ZEROS
ARE CREATED
EQUAL
0 CALORIE COLA
pepsi MAX
zero calories
maximum taste
ZERO CALORIES. MAXIMUM PEPSI® TASTE.
pepsi MAX
zero calories

Two's Complement Numbers



Really neat trick: make the most significant bit represent a *negative* number instead of positive:

$1101_2 = -8 + 4 + 1 = -3$

$1111_2 = -8 + 4 + 2 + 1 = -1$

$0111_2 = 4 + 2 + 1 = 7$

$1000_2 = -8$

Easy addition: just add in binary and discard any carry.

Negation: complement each bit (as in one's complement) then add 1.

Very good property: no -0

Two's complement numbers are equal if all their bits are the same.

Number Representations Compared

Bits	Binary	Signed Mag.	One's Comp.	Two's Comp.
0000	0	0	0	0
0001	1	1	1	1
⋮				
0111	7	7	7	7
1000	8	−0	−7	−8
1001	9	−1	−6	−7
⋮				
1110	14	−6	−1	−2
1111	15	−7	−0	−1

Smallest number

Largest number

Fixed-point Numbers

How to represent fractional numbers? In decimal, we continue with negative powers of 10:

$$\begin{aligned} 31.4159 &= 3 \times 10^1 + 1 \times 10^0 + \\ &\quad 4 \times 10^{-1} + 1 \times 10^{-2} + 5 \times 10^{-3} + 9 \times 10^{-4} \end{aligned}$$

The same trick works in binary:

$$\begin{aligned} 1011.0110_2 &= 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 + \\ &\quad 0 \times 2^{-1} + 1 \times 2^{-2} + 1 \times 2^{-3} + 0 \times 2^{-4} \\ &= 8 + 2 + 1 + 0.25 + 0.125 \\ &= 11.375 \end{aligned}$$

Need a bigger range? Try Floating Point Representation.

Floating point can represent very large numbers in a compact way.

A lot like scientific notation, -7.776×10^3, where you have the *mantissa* (-7.776) and *exponent* (3).

But for this course, think in binary: $-1.10x2^{0111}$

The bits of a 32-bit word are separated into fields. The IEEE 754 standard specifies

- which bits represent which fields (bit 31 is sign, bits 30-23 are 8-bit exponent, bits 22-00 are 23-bit fraction)
- how to interpret each field