CSEE W3827

Fundamentals of Computer Systems

Homework Assignment 1

Prof. Martha A. Kim

Columbia University

Due September 19, 2013 at 10:10 AM

Write your name **and UNI** on your solutions

Show your work for each problem; we are more interested in how you get the answer than whether you get the right answer.

1. (5 pts.) What are the values, in decimal, of the bytes

   11011011

   and

   01011001,

   if they are interpreted as 8-bit

   (a) binary numbers;

   (b) two's complement numbers?

2. (10 pts.) Show how to compute $47 + 55$ in *binary*.

3. (10 pts.) Complete the truth table for the following Boolean functions:

$$a = XY\overline{Z} + \overline{Y}Z + \overline{X}\,\overline{Z}$$
$$b = (X + \overline{Z})(\overline{Z} + Y)(\overline{Y} + \overline{X})$$

X	Y	Z	a	b
0	0	0		
0	0	1		
0	1	0		
0	1	1		
1	0	0		
1	0	1		
1	1	0		
1	1	1		

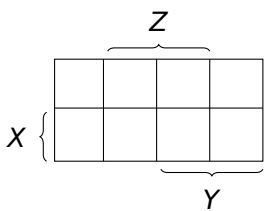
4. (15 pts.) Consider the function F, whose truth table is below.

X	Y	Z	F
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

(a) Write F as a sum of minterms.

(b) Write F as a product of maxterms.

(c) Complete the Karnaugh map for F as shown below.



(d) Give a simplified expression for F.

(e) Draw a schematic for F.

5. (20 pts.) For each pair of A and B below, prove either that $A = B$ or $A \neq B$.

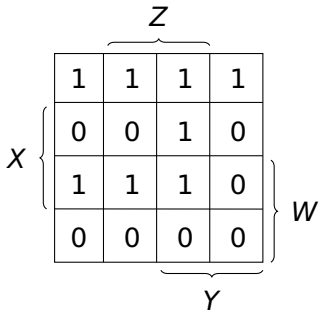
(a)

$$A = \overline{X}\,\overline{Y}Z + \overline{X}Y\overline{Z} + X\overline{Y}\,\overline{Z} + XYZ$$
$$B = (X + Y + Z)(X + \overline{Y} + \overline{Z})(\overline{X} + Y + \overline{Z})(\overline{X} + \overline{Y} + Z)$$

(b)

$$A = X + \overline{Y}Z + Y\overline{Z}$$
$$B = XZ + \overline{Y}Z + \overline{X}\,\overline{Y}Z + X\overline{Z}$$

6. (5 pts.) Give a minimal boolean expression for the function below.



7. (20 pts.) Design a circuit that takes one four-bit binary number (X_3, X_2, X_1, X_0) and produces the following outputs:

   - EV: one bit indicating whether or not X is even
   - PR: one bit indicating whether or not X is prime
   - $P2$: one bit indicating whether or not X is a power of two
   - SQ: one bit indicating whether or not X is a square
   - RT_1,RT_0: two bits indicating the square root of X if X is a square

(a) Fill in the rest of the truth table.

X_3	X_2	X_1	X_0	EV	PR	$P2$	SQ	RT_1RT_0
0	0	0	0	1	0	0	1	00
0	0	0	1	0	0	1	1	01
0	0	1	0	1	1	1	0	XX
0	0	1	1					
0	1	0	0					
0	1	0	1					
0	1	1	0					
0	1	1	1					
1	0	0	0					
1	0	0	1					
1	0	1	0					
1	0	1	1					
1	1	0	0					
1	1	0	1					
1	1	1	0					
1	1	1	1					

(b) Give minimal boolean expressions for EV, PR, $P2$, SQ, RT_1 and RT_0.

(c) Draw the circuit for EV, PR, $P2$, SQ, RT_1 and RT_0. No need to optimize across outputs, unless you wish to.

8. (15 pts.)

(a) Minimize the function $F = \overline{A}\,\overline{B}\,\overline{C} + \overline{A}\,\overline{B}\,C + A\,\overline{B}\,\overline{C} + A\,\overline{B}\,C + A\,B\,\overline{C}$.

(b) Use bubble pushing to implement F using only NAND2 gate(s) and inverter(s).