Throughput and Energy Efficiency in Topology-Controlled Multi-hop Wireless Sensor Networks

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ABSTRACT

In the context of multi-hop wireless networks, various topology control algorithms have been proposed to adapt the transmission range of nodes based on local information while maintaining a connected topology. These algorithms are particularly suited for deployment in sensor networks which typically consist of energy constrained sensors. Sensor nodes should support power adaptation in order to use the benefits of topology control for energy conservation. In this paper, we design a framework for evaluating the performance of topology control algorithms using overall network throughput, and total energy consumption per packet delivered, as the metrics. Our goal is to identify the scenarios in which topology control improves the network performance. We supplement our analysis with *ns2* simulations using the cone-based topology control algorithm [10, 19].

Based on our analysis and simulations, we find that link layer retransmissions are essential with topology control to avoid throughput degradation due to increase in number of hops in lightly loaded networks. In heavily loaded networks, the throughput can be improved by a factor up to k^2, where k is the average factor of reduction in transmission range using topology control. Studies of energy consumption reveal that improvements of up to k^4 can be obtained using topology control. However, these improvements decrease as the traffic pattern shifts from local (few hop connections) to non-local (hop lengths of the order of the diameter of the network). These results can be used to guide the deployment of topology control algorithms in sensor networks.

Categories and Subject Descriptors

C.2.1 [**Network Architecture and Design**]: Network Communications, Network Topology, Wireless Communication

General Terms

Performance, Experimentation

WSNA'03, September 19, 2003, San Diego, California, USA.

Keywords

Sensor Networks, Ad-hoc Networks, Wireless Networks, Topology Control

1. INTRODUCTION

Multi-hop wireless networks, such as radio networks [7], ad-hoc networks [14], and sensor networks [4, 15], are networks where communication between two devices may go through multiple consecutive wireless links. Unlike wired networks for which the topology is fixed (except in cases of link failures), these networks have dynamic topologies based on the location of the devices and their range of transmission. For sufficiently dense networks, nodes can restrict communication only to nearby nodes with reduced range of transmission and yet maintain global network connectivity. The mechanism for computing the sufficient transmission ranges for each node is called *topology control*, and its main goal is to optimize performance metrics such as network lifetime and throughput while maintaining a connected network.

Reduction in transmission range reduces the per packet energy consumption and also promotes spatial reuse. However lengthening of routes effects the end-to-end reliability of flows. Taking these factors into account, it becomes critical to understand the applicability and usefulness of topology control in various networking scenarios. To this end, the goal in this paper is to design a framework for analyzing the performance of networks with topology control and to identify scenarios where topology control is beneficial to use. Our study is focused on UDP flows and the metrics used are throughput improvement, and energy efficiency. We define throughput to be the total number of bits received per second by the destinations of all the multi-hop flows in the network. For purposes of energy consumption comparisons, the metric we use is the total transmission and reception energy consumed per successfully received bit.

This paper makes the following two contributions to the understanding of the performance of topology control. First, we present an analytical framework for characterizing the performance of topology control under different traffic patterns, packet error rates, and maximum number of link layer retransmissions. Second, using the cone-based topology control algorithm [10] as an example, we show that the results from realistic simulation setting closely follow our analysis. Based on our studies, the performance of topology control for large networks can be characterized as follows:

- In heavily loaded scenarios, topology control can improve the throughput by a factor up to k^2, where k is the average factor of reduction in transmission range obtained by using topology control.

- In lightly loaded scenarios, channel error causes reduction in throughput. Increase in link layer persistence lessens the severity of throughput reduction.
- Studies of energy consumption reveal that improvements of up to k^4 can be obtained using topology control for local unicast traffic. For an unreliable link layer, Topology Control increases the energy consumption, if the traffic pattern is non-local. However, this can be effectively prevented by the use of link layer retransmissions.

While the results in this paper apply to wireless ad hoc networks in general, the following two main aspects of the framework are specifically targeted for sensor networks. First, we assume a network of relatively large number of nodes that are randomly placed in a region. The random distribution can be a result of sensor deployment from moving aircrafts or vehicles. Second, we consider energy efficiency as one of the study parameters, which is vital for sensor networks, as replacing or recharging batteries is difficult, if not impossible.

The rest of the paper is organized as follows. Section 2 characterizes the benefits of using topology control in terms of end-to-end throughput improvements and energy efficiency. Section 3 presents results from simulation on the *ns2* simulator using an idealized MAC layer and a simple fixed error rate channel model as well as a more realistic ambient noise dependent channel model. In Section 4, we present related work in the field of topology control for multi-hop wireless networks. Section 5 points to areas which are part of our future work. Section 6 concludes the paper.

2. ANALYSIS OF NETWORK THROUGHPUT AND ENERGY EFFICIENCY

In this section we analyze the performance of networks with topology control in terms of network throughput and energy efficiency. We first state our assumptions and describe our traffic model. We then analyze the performance of networks using topology control under different traffic patterns, network loads, link layer reliability and channel error rates.

2.1 Assumptions and Traffic Model

The range of interference for packet transmissions is larger than the transmission range. For our analysis, we assume that within the range of interference, no other transmissions or receptions may occur simultaneously[1]. Nodes closer to the boundary of the network may have part of their interference region outside the network boundaries. In order to properly apply our analytical results to the simulation scenarios, we define the notion of *effective interference range.* For a node with its interference region completely inside the network boundary, the effective interference range is the same as the interference range. Otherwise, it is defined as the radius of a disk whose area is equal to the area of the interference region inside the network boundaries. We refer to the scheme where no topology control is used as *Plain.* Let R_{tc} and r_{tc} be the average effective interference range and transmission range respectively, when topology control is used and let R_{pl} and r_{pl} be the corresponding notations for Plain. We assume the network is dense enough such that the network is connected if every node uses r_{pl} as its transmission range. Let $k_e = \frac{R_{pl}}{R_{tc}}$ and $k = \frac{r_{pl}}{r_{tc}}$. If the network is sufficiently large, $k_e = k$ as typically the interference range scales with the transmission range. Throughout this paper, the subscript tc is used to refer to a parameter associated with the network using Topology Control, and the subscript pl to refer to the parameter associated with the network without using Topology Control. We drop the subscript tc and pl when our discussion applies to both. If R_m is the maximum transmission range, then $R_{pl} = R_m$. We assume that the network nodes are uniformly distributed in a square area of size A.

The bit error rate is a function of various parameters including the modulation scheme, the received power, the ambient noise and the raw channel bit-rate. For our analysis we make the simplifying assumption that the channel error rate is independent of the received power. However, in our simulations we consider both, a realistic channel error model based on received power and ambient noise (see Section 3.3), and the error model independent of received power. The former model considers our results in a more realistic setting while the latter provides insights into our study as it corresponds to the model used in our analysis.

We assume that all packets are of same length. Our analysis assumes that the transmission range is uniform across all the nodes in the network[2]. This average transmission range will be smaller for Topology Control than for Plain. We do not model the possible effects of routing protocols and MAC layer collision avoidance mechanisms. We leave it to the simulations to explore these dimensions of the problem.

We use the power law traffic pattern as our traffic model [8]. It allows modeling of a variety of traffic patterns ranging from "local" traffic to traffic spanning the diameter of the network. Under the power law distribution of network traffic, the probability that a node communicates with a destination at a distance of x, is proportional to x^λ, for a constant λ. So, the expression for the probability is given by

$$q(x) = \frac{x^\lambda}{\int_\epsilon^{\sqrt{A}} t^\lambda dt} \tag{1}$$

ϵ is introduced to make the probability density function well-defined for $\lambda \leq -1$. It is a non-zero minimum distance between any source-destination pair. The power law distribution captures the overall locality of network traffic. For large negative λ, destinations are clustered close to the sender. For large positive λ, destinations are dispersed to the periphery of the network. $\lambda = 0$ corresponds to a traffic pattern where the distance between the source and the destination is picked from a uniform distribution.

For this traffic pattern, the average path length L is given by

$$L = \int_\epsilon^{\sqrt{A}} x \frac{x^\lambda}{\int_\epsilon^{\sqrt{A}} t^\lambda dt} dx \tag{2}$$

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[1] A more realistic interference model is used in our simulations.

[2] This assumption holds well when the node density is uniform.

Closed form expression of L with respect to different λ can be found in [8]. Given this traffic pattern with various values of λ, we derive expressions for the throughput ratio and energy efficiency ratio of Topology Control over Plain. We present the expressions graphically based on numerical solutions obtained using the Matlab toolkit.

2.2 Throughput Analysis

We study two different types of network scenarios, namely saturated and unsaturated. The distinction is based on the total MAC layer traffic load in the network. In a saturated network, the MAC layer can not handle any extra transmissions beyond the current load. For such scenarios, besides other factors, the total end-to-end throughput is limited by the hop-by-hop throughput, commonly referred to as the capacity of the network. In an unsaturated network, the raw channel capacity is not fully utilized. For such scenarios, the limiting network capacity will not play any role. However, other factors such as channel error rate and maximum number of allowed link layer retransmissions impact the throughput ratio. We therefore study the two cases separately.

2.2.1 Unsaturated Networks

In this scenario, the shared media has very few packets to send. Let Θ be the total end-to-end traffic that gets injected into the network per unit time. This does not include packets that have not left the sender's buffer. Let $S(x)$ be the probability that a packet will be delivered to a destination at a distance of x units. Clearly $S(x) = (1 - p^{mtx})^{h(x)}$ where p is the probability of transmission failure on a link, mtx is the maximum number of transmission attempts made by the link layer, and $h(x)$ is the number of hops needed for the packet.

In a highly dense network, $h(x)$ can be approximated by $\lceil \frac{x}{r} \rceil$ (r is the average transmission range). If the network density is not very high, the expression for $h(x)$ is a lower bound on the hop length. Since the total traffic for distance x is $\Theta q(x)$, the total traffic successfully delivered to distance x is $\Theta q(x)S(x)$. The overall throughput integrated over all distances is thus,

$$T = \int_{\epsilon}^{\sqrt{A}} \Theta q(x)S(x)dx \tag{3}$$

In unsaturated scenarios, the total traffic that the senders are generating is exactly the traffic that gets injected into the network (Θ). Therefore Θ will drop out of the equation when we consider the throughput ratio, which is given by

$$\begin{aligned} T_{ratio} &= \frac{T_{tc}}{T_{pl}} \\ &= \begin{cases} 1, & \text{if p=0} \\ \frac{1-(1-p^{mtx})^{\frac{\sqrt{A}}{r_{tc}}}}{1-(1-p^{mtx})^{\frac{\sqrt{A}}{r_{pl}}}}, & \text{if } \lambda\text{=0} \\ \cdots \end{cases} \end{aligned} \tag{4}$$

Since the closed form solution for general p and λ is cumbersome and unwieldy, we only give the closed form solution for specific cases. We resort to plots from numerical solutions to illustrate the value of T_{ratio} for different p and λ. Clearly, the overall throughput will be the same for both Plain and Topology Control provided that the link is reliable. Since normally $r_{tc} < r_{pl}$, Equation 4 shows that the throughput of Topology Control will be less than Plain. We plot the throughput ratio in Figures 1(a) and 1(b). The parameters used are as follows: $r_{pl} = 250m$, $r_{tc} = 92m$ and $\rho = 1.56 * 10^{-4}$ nodes per square meter. The transmission range r_{pl} is based on Lucent WaveLAN's transmission range that is also used in our simulations. The transmission range for Topology Control r_{tc}, is based on the average transmission range observed in our simulations with a node density of ρ. Figure 1(a) shows that the throughput of Topology Control will decrease drastically as λ and p increases. Since link layer retransmissions increase $S(x)$, the throughput ratio will decrease gradually with increasing λ if link layer retransmits as shown in Figure 1(b).

Intuitively, in an unsaturated network, the reduction in spectrum utilization does not improve the throughput as the spectrum is not the bottleneck. The end-to-end reliability of the packets impact the total throughput.

2.2.2 Saturated Networks

The throughput in this case will be the same as in Equation 3 except that Θ will no longer be same for Topology Control and Plain. Θ is a function of p, number of link layer transmission attempts mtx, and transmission range r. The proportion of traffic that can go into the air for a destination distance x away is $\Theta q(x)$. Let $G(x)$ denote the expected number of hop-by-hop transmissions that occur when a packet begins its journey to a destination at a distance x away (See appendix for the expression of $G(x)$). The total one hop transmissions per unit time will then be $\Omega = \int_{\epsilon}^{\sqrt{A}} \Theta q(x)G(x)dx$. If the network is saturated, then the maximum number of possible one hop transmissions per unit time is $\frac{WA}{\pi R^2}$ where W is the raw bandwidth of the channel, R is the effective sensing range. Let η be the efficiency of the MAC layer. Then $\Omega = \eta \frac{WA}{\pi R^2}$. Therefore,

$$\Theta = \frac{\eta WA}{\pi R^2 \int_{\epsilon}^{\sqrt{A}} q(x)G(x)dx} \tag{5}$$

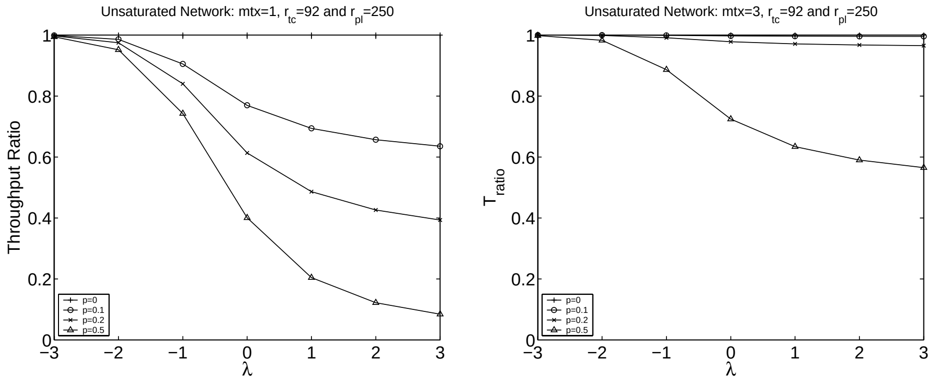
Thus, the throughput ratio is,

$$\begin{aligned} T_{ratio} &= \frac{T_{tc}}{T_{pl}} \\ &= \begin{cases} k_e^2 \frac{max(1, \frac{L}{r_{pl}})}{max(1, \frac{L}{r_{tc}})}, & \text{if p=0} \\ \cdots \end{cases} \end{aligned} \tag{6}$$

From Equation 6 and Figure 2, we see that Topology Control can achieve a throughput k_e^2 times more than Plain if the traffic pattern is local. As the traffic pattern becomes more and more non-local, the throughput ratio drops.

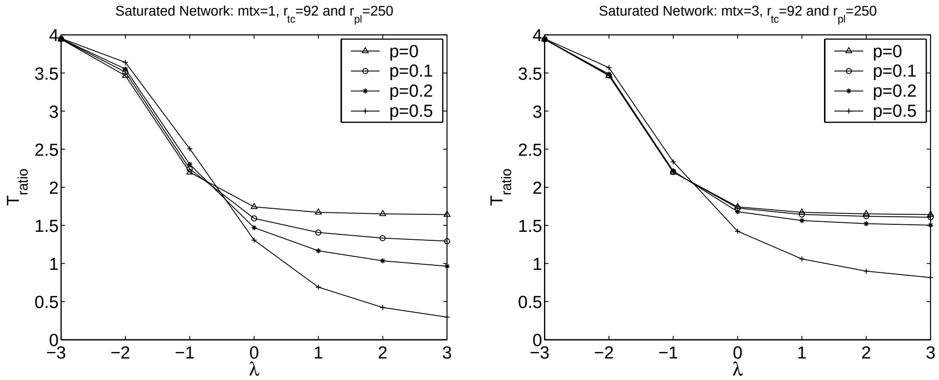
Note that, in the above analysis we assume that for Topology Control, the variance of the transmission and interference ranges across nodes in the network is small. Otherwise, T_{ratio} can be much smaller.

Intuitively, in a saturated network, the extra available spectrum obtained due to topology control, is used up by other flows as the spectrum is the bottleneck. As a result, for saturated networks, topology control improves the throughput, specially for local traffic. For non-local traffic, the end-to-end reliability decreases sharply for the case of topology control, and may reduce the total throughput.



(a) No link layer retransmission (b) Up to two retransmissions

Figure 1: Unsaturated networks: throughput ratio T_{ratio} with respect to λ and packet dropping probability p.



(a) No link layer retransmission (b) Up to two retransmissions

Figure 2: Saturated networks: throughput ratio T_{ratio} with respect to λ and packet dropping probability p.

2.3 Energy Efficiency

Recall that energy efficiency is defined to be the ratio of the amount of energy consumed per successfully delivered packet for Plain over that of Topology Control. Therefore, by definition, energy efficiency is independent of network load.

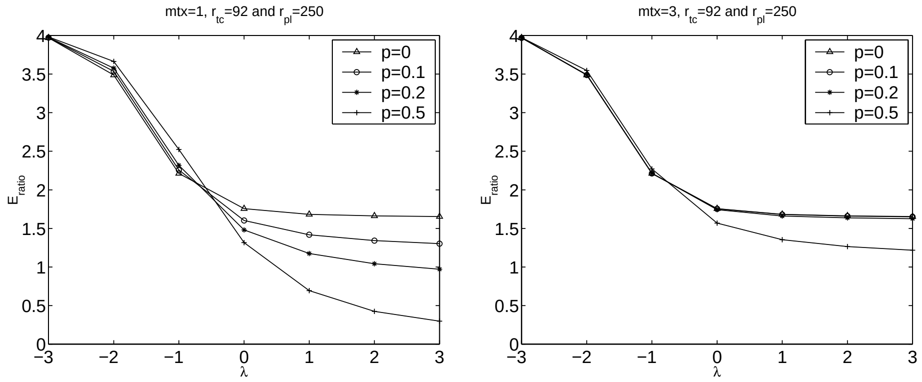
For Plain, the transmission energy E_{pl}^{tx} is proportional to R^{γ}, where both R, the maximum transmission range, and γ, the power loss factor are constants. So, E_{pl}^{tx} is a constant. However, for Topology Control, E_{tc}^{tx} is proportional to r^{γ}, where the range r is discovered as a result of the topology control algorithm. We use $\gamma = 4$ for our studies. Now we compute the energy consumed for receiving packets. For Topology Control, every packet transmission is received by $\pi R^2 \rho - 1$ nodes, where ρ is the density of nodes in the network. Each packet reception consumes a constant energy E^{rx}. The sum of transmission energy and receiving energy associated with one transmission is then $E = E^{tx} + (\pi R^2 \rho - 1)E^{rx}$. The total energy consumption for a packet trying to reach a destination at a distance of x is $G(x)E$. Therefore, the total energy consumption is $E_{total} = \int_{\epsilon}^{\sqrt{A}} \Theta q(x) G(x) dx$. Since the throughput T measures the number of packets delivered per unit time, the energy consumption per packet delivered E^{ef} is proportional to $\frac{E_{total}}{T}$. Thus the energy efficiency ratio is

$$\begin{aligned} E_{ratio} &= \frac{E_{pl}^{ef}}{E_{tc}^{ef}} \\ &= \frac{\int_{\epsilon}^{\sqrt{A}} q(x) S_{tc}(x) dx}{\int_{\epsilon}^{\sqrt{A}} q(x) S_{pl}(x) dx} \times \frac{\int_{\epsilon}^{\sqrt{A}} q(x) G_{pl}(x) E_{pl} dx}{\int_{\epsilon}^{\sqrt{A}} q(x) G_{tc}(x) E_{tc} dx} \\ &= \begin{cases} \frac{max(1, \frac{L}{r_{pl}}) E_{pl}}{max(1, \frac{L}{r_{tc}}) E_{tc}}, & \text{if p=0} \\ \ldots \end{cases} \end{aligned} \tag{7}$$

If we consider transmission power alone, then E_{ratio} can be up to k^4. If we consider reception power only, then E_{ratio} can be up to k_e^2. In order to illustrate the energy efficiency, we plot it using the following parameters, $r_{pl} = 250$, $r_{tc} = 92$ $\rho = 1.56 * 10^{-4}$ nodes per square meter, and $E_{tc}^{rx} = E_{pl}^{rx} = \frac{1}{2} E_{pl}^{tx}$.

From Figure 3, we see that Topology Control saves energy when the traffic pattern is local. Link layer retransmission improves only a little bit.

We observe that for traffic pattern ranging from local to random ($\lambda = 0$ corresponds to traffic where the distance to the destination is picked uniformly), Topology Control



(a) No link layer retransmission (b) Up to two retransmissions

Figure 3: Energy efficiency E_{ratio} with respect to λ and packet dropping probability p.

is always better in terms of energy efficiency. However, for unreliable channel model, energy efficiency of Topology Control is lower when the traffic pattern is to destinations far away ($\lambda > 0$). The energy efficiency can be improved by link layer retransmissions.

3. SIMULATION RESULTS

In our analysis in Section 2, we made some simplifying assumptions (Section 2.1). The goal of our simulations is to study the performance of Topology Control and Plain in realistic settings. We find that the conclusions from our analysis are similar to those from the simulations. And, in most cases the results from the simulations match closely with the results from the analysis.

3.1 Simulation Setup

In order for topology control to be practical, the algorithm must be distributed and use only local information, such as the algorithms in [10, 17, 9] ([9] is an improvement over [17]). Li et al. show that [10] and [9] produce similar topologies. We expect the results to be similar for the two algorithms [10, 9], and chose to simulate the algorithm in [10].

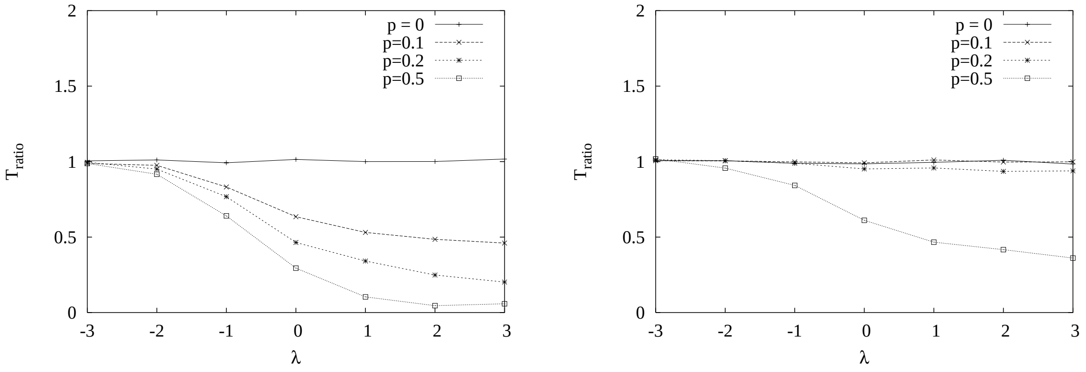
The algorithm is implemented in ns-2 network simulator [16], using the wireless extension developed at Carnegie Mellon [6]. We implement the CBTC algorithm proposed in [10] with $\alpha = 2\pi/3$ and its optimizations. According to the algorithm, each node independently picks the minimum transmission range such that for every cone of degree α from that node, there is a neighbor in the reduced transmission range. In practice, frequent message exchanges are needed to maintain these neighbors which can incur a significant overhead. We implemented asymmetric edge removal, shrink-back and pair-wise edge removal optimizations as discussed in [10]. Since our goal is not to study a specific topology control algorithm but to characterize the performance of topology control algorithms, we assume perfect knowledge of the location (x-y coordinates) of neighbors. This corresponds to a long-lived static network where neighbors do not go down or go away. For the same reason, we also pre-compute shortest routes based on global information.

The standard IEEE 802.11 does not work correctly with topology control algorithms. The reason is that the mechanism in IEEE 802.11 that deals with the hidden-terminal problem (RTS/CTS/ACK) assumes that every node has the same transmission range. This is not the case for networks using topology control, as each node determines its own transmission range. Several power efficient protocols [20, 13] that work in the presence of non uniform transmission ranges of nodes have been proposed. When used in conjunction with topology control, these MAC layer solutions will further improve energy efficiency and throughput. However, to keep our study independent of any MAC layer behavior we use an idealized version of a CSMA/CA protocol using knowledge of transmissions in the nearby region. For this purpose, we choose to modify the WaveLAN-I [18] CSMA/CA MAC protocol. Specifically, a transmission will not proceed if it can collide with any receiving packet at other nodes (i.e. the media is considered busy in that case) or if the packet will collide at the receiver. We refer to this MAC layer as the IDEAL MAC layer. Note that, the busy tone in PCMA [13] can achieve this effect. But the use of IDEAL MAC may underestimate the collision rate that would occur on a real MAC.

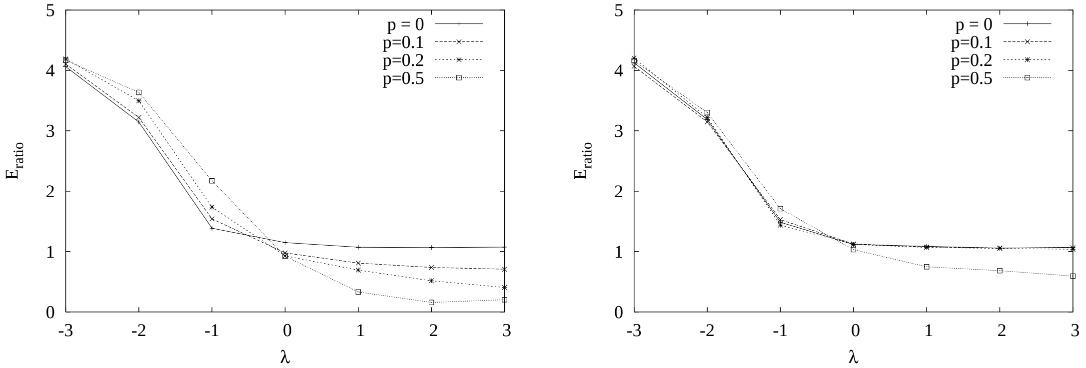
We assume that the received power drops with distance with an exponent of 4. That is, for the same received power, the transmit power is proportional to d^4, where d is the transmission distance. We assume that the power required to receive a transmission at the receiver is a constant and is one-half the maximum transmit power. The carrier frequency is 914MHz, and the transmission raw bandwidth 2MHz. We assume omni-directional antennas with 0dB gain, and the antenna is placed 1.5 meters above a node. The receive threshold is -94 dBW. The carrier sense threshold is -108 dBW and the capture threshold is 10 dB. These parameters simulate the 914 MHz Lucent WaveLAN DSSS radio interface. The receiving power is 0.1409 W.

Each node has a maximum transmission range of 250 meters which corresponds to a transmission power of 0.2818 W. There have been studies on network lifetime by assuming that each node has a fixed amount of energy supply [10, 3]. We focus on the performance of the network before any node runs out of energy. The nodes are placed uniformly at random in a square region of 800 by 800 meters.

For our simulations we have used two different link error models. The first model is a simple error model that corresponds to the one used in our analysis. In this model, the



(a) T_{ratio} with no retx. (b) T_{ratio} with max. retx. $= 2$



(c) E_{ratio} with no retx. (d) E_{ratio} with max. retx. $= 2$

Figure 4: 100 node unsaturated network: performance with channel error independent of transmission power (Comparable analytical results shown in Figures 1 and 3)

channel error rate is represented by a fixed probability. In our second model, the bit error rate is a function of several factors such as the ambient noise, the received power and the raw bit rate. The model also depends on the modulation scheme. This model helps in studying the performance of Topology Control more realistically. In the remaining section, we present the results based on the two error models.

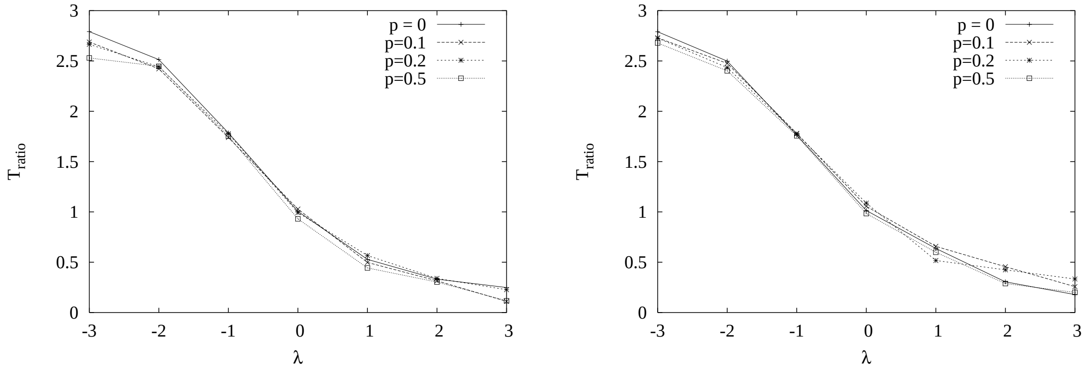
3.2 Simulation Results: fixed link error model

We vary the traffic pattern and show how it affects the performance of Topology Control over Plain. We consider the class of traffic patterns with power law distance distribution.

We simulate a 100 node network deployed in the 800m x 800m square region. The measured average transmission range of Topology Control is $r_{tc_1} = 92m$. Since r_{pl} is 250m, the transmission range ratio is $k_1 = \frac{r_{pl}}{r_{tc_1}} = \frac{250}{92} = 2.72$. The interference range of Plain is $550m$. For Plain, the measured average effective interference range R_{pl} is $390m$. For Topology Control, the measured average effective interference range $R_{tc1} = 187.7$. Therefore, $k_{e1} = 2.04$, $k_{e2} = 2.80$.

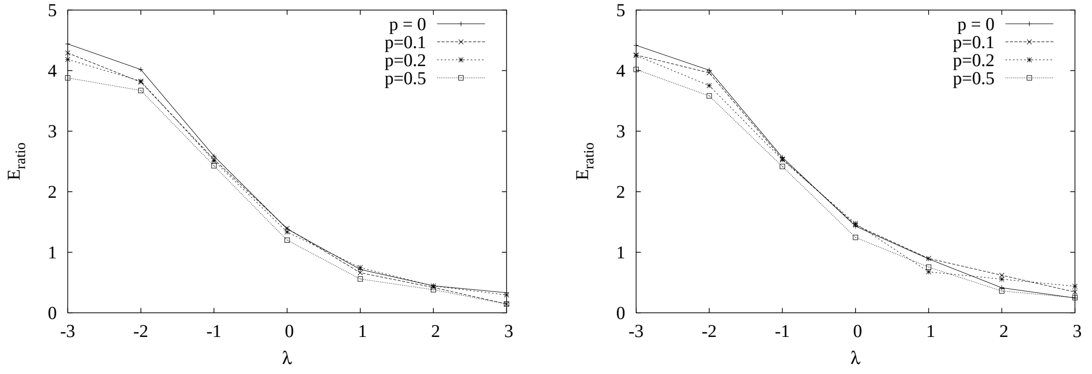
First, we present results for the unsaturated scenario. We plot the simulation results on throughput ratio and energy efficiency ratio with respect to different λ and p in Figure 4. For throughput ratio, the simulation graphs of Figures 4 (a) and 4(b) correspond to the analysis graphs of Figures 1(a) and 1(b). We observe that although the simulation results are slightly lower than the throughput ratios obtained from analysis, the pattern of the corresponding graphs are very similar. Similarly for energy efficiency, the simulation graphs of Figures 4 (c) and 4(d) correspond to the analysis graphs of Figures 3(a) and 3(b). Once again the patterns from the analysis match very well with the simulations. However, for values of $\lambda \geq 0$, the energy efficiency observed in simulations is lower than the results obtained from analysis. The main reason for this discrepancy is the approximate model used in our analysis that assumes uniform transmission power across all nodes in the network when topology control is used. In the simulations, the nodes have varying transmission ranges depending on the result of applying the topology control algorithm on the local topology. We observe several interesting facts from these graphs: (1) Even modest retransmissions such as 2 times significantly increases the throughput of Topology Control. (2) Energy efficiency ratio decreases drastically as λ increases. Energy efficiency ratio is only slightly affected by p and link layer retransmissions.

We present the results of the saturated case in Figure 5. As our analysis predicts, neither the packet error rate nor the number of link layer retransmissions affect the throughput ratio very much. Note that the throughput ratio at $\lambda = -3$ and $p = 0$ is around 2.8 (Figure 5(a)). which is less than the $k_{e1}^2 = 4$. This is the effect of the variance of transmission range of Topology Control. Also observe that the pattern of the graphs in Figure 5 corresponds very closely to our analysis graphs in Figure 2 and Figure 3. Like



(a) T_{ratio} with no retx.

(b) T_{ratio} with max. retx. = 2



(c) E_{ratio} with no retx.

(d) E_{ratio} with max. retx. = 2

Figure 5: 100 node saturated network: performance with channel error independent of transmission power (Comparable analytical results shown in Figures 2 and 3)

in the unsaturated case, the energy efficiency graph shows that Topology Control does not save energy if the traffic pattern is non-local.

3.3 Simulation Results: ambient noise based link error model

Our analysis as well as the simulations presented thus far assumed that the channel error rate is independent of the received power. However, in reality the bit error rate (p_b) is a function of the modulation scheme, the received power, the ambient noise and the raw channel bit rate. Assuming BPSK (binary-phase-shift-keying) modulation [3], the expression for bit error rate p_b, is given by [5]:

$$p_b = 0.5 * erfc(\sqrt{\frac{P_r}{N * f}}) \tag{8}$$

where N is the noise spectral density (noise power per Hz), f is the raw channel bit rate which for our simulations is 2 Mbps, and $erfc(x)$ is given by:

$$erfc(x) = 1 - \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt \tag{9}$$

We have studied our results under this channel model as well, and have found the same pattern in terms of throughput ratio and energy efficiency ratio, as observed in our analysis and previous simulations with simplified received power independent channel error model. We present a few results for this channel model and show conformance with previous observations.

We present the throughput ratios for both saturated and unsaturated networks with 100 nodes. Figure 6(a) shows the unsaturated scenario without any retransmissions. As observed in our analysis and simulation with simple channel model, under low network load, the throughput drops significantly with increase in the ambient noise level N. The ambient noise level effects the packet drop probability. By adding retransmissions the throughput ratio can be improved. Figure 6(b) shows the unsaturated scenario without any retransmissions. Once again our observations are very similar to the simple channel model and analysis. The same holds for energy efficiency (not presented here due to lack of space). Thus we observe that the performance characterization of topology control using the simple channel model is applicable to the more realistic channel model as well.

4. RELATED WORK

Recent work on topology control has concentrated on finding algorithms to improve throughput and energy efficiency while guaranteeing global connectivity using only local position or direction information. In the cone-based topology control algorithm (CBTC) [19, 10] Li Li et al. have studied,

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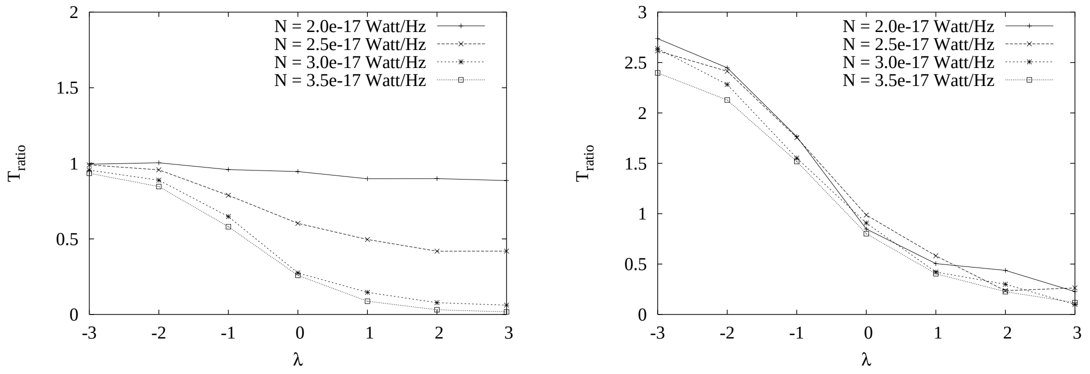
[3] For modulation schemes OOK and M-FSK an extra constant inside the square root is needed for the expression of p_b



(a) T_{ratio} with no retx. (Unsaturated Network)

(b) T_{ratio} with no retx. (Saturated Network)

Figure 6: With noise dependent channel error model: 100 node network

a node u transmits with the minimum power $p_{u,\alpha}$ required to ensure that in every cone of degree α around u, there is some node that u can reach with power $p_{u,\alpha}$. They showed that taking $\alpha = 5\pi/6$ is a necessary and sufficient condition to guarantee that network connectivity is preserved. More precisely, if there is a path from s to t when every node communicates at maximum power then, if $\alpha \leq 5\pi/6$, there is still a path in the smallest symmetric graph G_α containing all edges (u, v) such that u can communicate with v using power $p_{u,\alpha}$. On the other hand, if $\alpha > 5\pi/6$, connectivity is not necessarily preserved. Rodoplu and Meng [17] propose a distributed position-based topology control algorithm that preserves connectivity and minimum energy path. Their algorithm is improved by Li Li and Halpern [9]. Ning Li et al. [11] proposed a minimum spanning tree based topology control algorithm, where each node builds its local minimum spanning tree independently and only keeps on-tree nodes that are one-hop away as its neighbors in the final topology. Fault tolerance issues have also been considered recently. Bahramgiri et al. [1] have shown that, for the CBTC algorithm, it is necessary to have $\alpha \leq 2\pi/3(k-k \bmod 2)$ in order to tolerate k node failures or to have k node-disjoint paths between any two nodes in the network. XiangYang Li et al. [12] have shown that a variant of Yao graph can also preserve k node-disjoint paths. None of these algorithms have characterized the operating region of topology control under different traffic pattern, load condition and packet error rate.

Bansal et al. [2] have studied the effect of different fixed transmission radius on the throughput of TCP traffic and energy efficiency. They have shown that the total energy consumption is a convex function of the transmission range due to tradeoffs between individual packet transmission energy and the likelihood of retransmission. The TCP session throughput decreases supra-linearly with a decrease in the transmission range. However, they do not consider other traffic patterns or the effect of topology control.

5. DISCUSSION AND FUTURE WORK

We have made some simplifying assumptions in our analysis. As part of our future work, we are interested in refining the presented analytical framework by taking the following factors into account:

- We would like to incorporate the distribution of the transmission range of Topology Control into our analysis. Our current analysis assumes that the transmission range of all nodes is uniform, albeit reduced, even when Topology Control is used.

- We have assumed that a transmission reserves an area within the sensing range. However, in protocols with RTS/CTS/ACK, the reservation is the union of areas within the sensing range of source and destination. We would like to model this type of MAC as well.

- Although our simulations have shown that the simplified link error model suffices for the purposes of characterizing Topology Control, a more accurate analysis will be based on a realistic channel error model such as the one used in part of our simulations.

We would also like to enhance our simulation studies by incorporating the following:

- We have used an idealized version of CSMA protocol for the MAC layer. We would like to investigate the effect of a real MAC protocol such as PCMA [13].

- Mobility has not been considered in our study and it remains to be investigated how the relative performance of topology control gets affected when nodes are mobile.

- Our simulation focuses only on UDP traffic. The effect of different transmission ranges on TCP throughput has been studied in [2]. However, they have also assumed that all nodes have the same transmission range. It would be interesting to study how TCP throughput gets affected by variable transmission ranges if topology control is used.

6. CONCLUSION

In this paper, we study the performance gains of topology control using analysis and simulations in terms of throughput and energy efficiency under different traffic patterns, network load conditions, and packet error rates.

Using the power law traffic pattern for UDP flows, we have shown that topology control improves the overall end-to-end network throughput up to a factor of k_e^2 (where k_e is the ratio of effective interference range) in saturated scenarios when the traffic pattern is local. Beyond random traffic pattern, i.e., when traffic becomes more and more non-local, topology control degrades the performance in terms of throughput. Although link layer retransmissions can improve the throughput for topology control significantly for local traffic, its effect on non-local traffic is minimal. The energy efficiency of topology control is independent of network load, and can be up to k^4 where k is the average factor of reduction in transmission range by the use of topology control. It is not effected much by link layer retransmissions. The energy efficiency of topology control decreases as the traffic pattern changes from local to non-local.

Based on our analysis and simulations, we conclude that sensor networks can benefit from the use of topology control both in terms of throughput and energy consumption, if the traffic pattern in the network is anywhere from local to random.

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APPENDIX

In this appendix, we derive the expected number of hop-by-hop transmissions $G(x)$ for a packet destined for a node at a distance of x. Let p be the packet error rate for each hop. Let mtx be the maximum number of transmission attempts of the link layer.

Since a transmission fails over a given hop only when all mtx transmissions fail, the probability of packet transmission failure over a link is $\bar{p} = p^{mtx}$. Given that the transmission of a single hop is successful (event χ), the conditional probability that the i-th attempt is successful is given by $Prob(i|\text{event } \chi) = \frac{p^{i-1}(1-p)}{1-p^{mtx}}$. Therefore, the conditional expected number of transmissions over a single link given event χ is $m_1 = \sum_{i=1}^{mtx} i \cdot Prob(i|\text{event } \chi) = \frac{1}{1-p} - \frac{mtx \cdot p^{mtx}}{1-p^{mtx}}$. Therefore the expected number of transmissions over a h hop path is $g(h) = \sum_{i=0}^{h-1} (m_1 * i + mtx) * (1-\bar{p})^i * \bar{p} + h * m_1 * (1-\bar{p})^h$. The expression $G(x)$ is simply $g(\lceil \frac{x}{r} \rceil)$ where r is the transmission range.