Throughput Optimization of Wireless Mesh Networks with MIMO Links

Randeep Bhatia Li (Erran) Li

Bell Laboratories, Lucent Technologies
{randeep, erranlli}@bell-labs.com

***Abstract*— Multiple Input Multiple Output (MIMO) antennas use sophisticated physical layer techniques to provide significant benefits over conventional antenna technology. Multiple independent data streams can be sent over the MIMO antenna elements. MIMO link can also suppress interference from neighboring links as long as the total useful streams and interfering streams are no greater than the number of receiving antenna elements. For these reasons MIMO antennas are increasingly being considered for use in interference limited wireless mesh networks and have been adopted by WLAN and WIMAX standards. However, the benefits of the MIMO technology in improving network performance are limited unless the higher layer protocols also exploit these capabilities.**

In this paper we are interested in characterizing the benefits of cross-layer optimizations in interference limited wireless mesh networks with MIMO links. We formulate a framework where data routing at the protocol layer, link scheduling at the MAC layer and stream control at the physical layer can be jointly optimized for throughput maximization in the presence of interference. We then develop an efficient algorithm to solve the resulting throughput optimization problem subject to fairness constraints.

I. Introduction

Wireless mesh networks are increasingly being deployed in commercial settings. For example, many US cities including Medford, Oregon [2] and Chaska, Minnesota [1] have deployed mesh networks. Even big cities like Philadelphia, San Francisco and Taipei are planning to deploy city-wide mesh networks to provide internet access to residents and local businesses. Such networks must provide high capacity while providing coverage over a large area in order to support high bandwidth applications such as video streaming and gaming etc. These desired goals are not easily achieved in interference limited mesh networks using conventional antenna technologies. In this context MIMO antenna technology holds tremendous promise. Although not completely standardized by the 802.11n working group, vendors such as Atheros are motivated to offer MIMO chip sets for use in commercially available wireless access points [5] and routers. The benefit of MIMO technologies comes from their ability to exploit rich scattering environment, possibly with significant multipath components, to reach their full potential. Thus, MIMO is ideal for dense urban and in-building environments which is precisely the deployment settings for mesh networks where the high data rates of MIMO are achieved. MIMO physical layer technologies utilize multiple antenna elements to create independent channels in these environment, thus achieving higher throughput.

A MIMO link is capable of different transmission strategies, each with different benefits. For instance in the spatial multiplexing strategy, link capacity is enhanced by sending independent data streams over the MIMO antenna elements. On the other hand, link reliability and communication range is enhanced by exploiting diversity when sending dependent data streams over the antenna elements. MIMO link can also suppress interference from neighboring links as long as the total useful streams and interfering streams are no greater than the number of receiving antenna elements. Thus, in order to realize the full potential of MIMO technology higher layers must be designed to be cognizant of the MIMO link capability. Indeed recent work in the literature have studied design of cross-layer aware MAC and routing protocols when using MIMO links [7], [6].

In this paper, our focus is on the more fundamental question of quantifying the potential realizable gains of MIMO links in cross-layer aware mesh networks. This is not a matter of simply translating the achievable gains for individual MIMO links into end-to-end gains due to the presence of interference limited channels and due to the multi-hop nature of the underlying network routing. On the other hand we develop a mathematical framework in which cross-layer gains can be expressed as a function of the network routing, link scheduling and stream control in the presence of interference. We then use this framework to formulate a network throughput optimization problem subject to fairness constraints on allocation of scarce wireless capacity among mobile users. As is expected the cross-layer throughput optimization problem proves challenging to solve optimally. We design an efficient algorithms to solve this optimization problem while also providing guarantees on the quality of the solution returned by the algorithm. To the best of our knowledge, ours is the first work to comprehensively characterize the benefit of MIMO in terms of end-to-end performance realized using cross-layer optimizations.

II. Network Architecture and Wireless Model with MIMO Links

In this section, we present background information on our network architecture and model.

A. Network Architecture

We study wireless mesh networks with MIMO links deployed in urban or residential setting. Figure 1 shows an

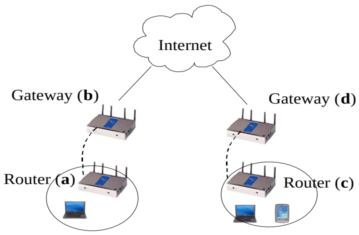

Fig. 1. A wireless Mesh network with 4 nodes. Two of them, b and d are gateways. Each node has 4 antenna elements. Dashed lines indicate node pairs are within transmission range. All four nodes are within interference range of one another.

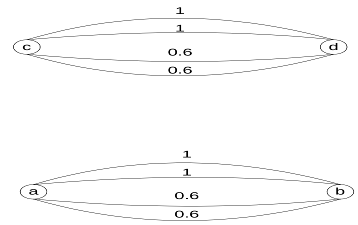

Fig. 2. Illustration of the need for stream control

example. These networks consist of wireless mesh routers and end mobile clients. Each wireless router has multiple antenna elements and can form MIMO links with neighboring wireless routers. Mobile clients may or may not have multiple antenna elements. Each mesh router is equipped with traffic aggregation capabilities (e.g. Access Points) and provide network connectivity to mobile clients within their coverage areas. The wireless mesh routers themselves form a multi-hop wireless backbone for relaying the traffic to and from the clients. Some of the wireless mesh routers are equipped with gateway functionality to enable connectivity to the wired Internet. All infrastructure resources that the mobile client access (e.g. web servers, enterprise servers, Internet gateways) reside on the wired Internet and can be accessed via any wireless mesh router with gateway functionality. Thus, the wireless backbone of mesh routers mainly relays mobile clients traffic to and from the Internet via the routers with gateway functionality.

B. MIMO links

MIMO links have the unique advantage of spatial multiplexing. As data is transmitted over a *matrix* rather than a vector channel, k independent data streams can be transmitted simultaneously over *eigenmodes* (or stream control) of a matrix channel if both transmitter and receiver are equipped with k antenna elements. The receiver can isolate and decode all k incoming streams successfully as long as the total number of streams is less than or equal to its number of antenna elements. A node with k antenna elements is also referred to as a node with k degree of freedoms (DOF). This is analogous to code-division multiple access (CDMA) transmission in which multiple users sharing the same time and frequency channel are mixed upon transmission and recovered through their unique codes. The key feature of MIMO is that no frequency spreading, hence no cost of spectrum efficiency is needed as the *spatial signature* of each signal is provided by nature in a multipath and rich scattering environment.

Because not all channel modes are equal, this translates into unequal gains to the streams using those modes. For example, the normalized gains of the four streams of the two MIMO link (a, b) and (c, d) of the network in Figure 1 can be 1, 1, 0.6 and 0.6 as shown in Figure 2. Since the two links interfere with each other, the total number of independent streams transmitted can not be greater than 4. If they transmit in a TDMA fashion, then the total normalized throughput is 3.2. However, if the best two channel modes are selected for transmission from each link, then the total normalized throughput is 4. How to select channel modes for transmission is also called *stream control*.

C. Wireless Transmission and Interference Model

We assume all communications use a common channel. For ease of description, we assume each node has the same number of K antenna elements. Two nodes can communicate with each other if they are within the transmission range of each other. We assume a uniform transmission range as we do not consider spatial diversity for range extension. We assume the total power used for transmission is the same for all nodes. We denote by R_T the *transmission range*. Denote $d(u, v)$ the distance between the nodes u and v. An edge $(u, v) \in E$ if and only if $d(u, v) \leq R_T$ and implies that mesh router u can communicate with mesh router v directly (in one hop). We denote by $c(e, i)$ the rate for edge $e = (u, v, i)$ transmitting i independent streams when the number of interfering streams at the receiver are no greater than $K - i$. Note that, $c(e, i)$ is independent of the location of the interfering sources. As shown in [3], it stays relatively the same as the distance of the interference source varies. Since MIMO link is half-duplex, a transmission will use the best i channel modes if i independent streams are transmitted. Note that, this applies to both closed-loop MIMO and MIMO with antenna selection [4]. We do not consider partial interference suppression [7] whereby fewer resources or degree of freedoms are needed if the interference is low. In this case, fewer than j DOFs can suppress j interfering streams. We note that in the 802.11 setting, the interfering range R_I is typically around two times R_T. According to results in [3], partial suppression does not perform well when the interfering sources are within R_I.

We denote by R_I the *interference range*. We assume that R_I is $q \times R_T$ where $q \geq 1$. We assume 802.11 media access control protocol is extended to allow simultaneous transmission within interference range, and that it prevents transmissions when the total streams at a receiver are greater than K. Such spatial multiplexing MAC has been developed [7] in the literature.

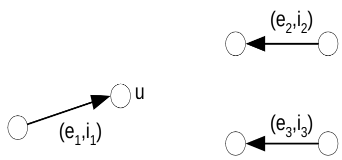

Fig. 3. Illustration for interference suppression: Here $e_2, e_3 \in I(e_1)$. Node u must have enough antennas not just to receive data on edge e_1 but also to suppress interference from the links e_2 and e_3. Thus for spatial multiplexing u must have at least $i_1 + i_2 + i_3$ antennas.

III. Problem Formulation and Algorithm Overview

Formally, we are given a wireless mesh backbone network modeled as a directed graph (V, E). Each node $u \in V$ is equipped with $I(u)$ antennas denoted by the set $A(u)$. The maximum number of antennas at a node is denoted by K. A link $e = (u, v) \in E$ denotes that direct communication is possible from node u to v. We assume that using MIMO technology upto K simultaneous data streams are possible on such a link e. Specifically if $k \leq K$ antennas are available at both the nodes u and v then upto k simultaneous data streams are possible. The rate for such a communication is a function of the two end nodes u and v and the number of streams k. We denote this rate by $c(u, v, k)$. In the following we will denote a communication on edge $e = (u, v)$ comprising of k simultaneous data streams by either (u, v, k) or (e, k). For such a communication $N(e, k) \geq k$ denotes the number of antennas used. If the link is using spatial multiplexing then $N(e, k) = k$. The physical model imposes send/receive constraint for communication. This means that a communication (u, v, k) cannot be accompanied by any other simultaneous communication on node u or v.

For each link $e = (u, v) \in E$ the set $I(e) \subseteq E$ denotes the set of links any of whose transmission causes interference to transmissions on link e (at node v). Simultaneous communication on links $e = (u, v)$ and a different link $e' = (u', v') \in I(e)$ is possible in our model (where u, v, u', v' are all distinct nodes). Specifically, communications (u, v, j) and (u', v', j') can take place simultaneously. However, in this case end-node v of link e must have at least $N(e, j) + N(e', j')$ antennas so that $N(e', j')$ DOF can be dedicated for interference suppression from link e'. In general for edge $e = (u, v)$ and for the communication (u, v, j) the node v must have at least $N(e, j) + N(e'_1, j'_1) + N(e'_2, j'_2) + N(e'_3, j'_3) + \ldots$ antennas, where links $e'_1, e'_2, e'_3, \ldots \in I(e)$ are simultaneously communicating with link e while using $j'_1, j'_2, j'_3 \ldots$ simultaneous data streams respectively. An example is shown in Figure 3.

A node u has an aggregated demand $l(u)$ from its associated users. We seek to maximize λ where at least $\lambda l(u)$ amount of throughput can be routed from each node u to the Internet (represented by a node t). In order to achieve $\lambda l(u)$ throughput for each node u we need to compute (1) a network flow that associates, for each link $e = (u, v) \in E$ and for each possible communication (e, i), the values $f(e, i)$. Here $f(e, i)$ or $f(u, v, i)$ denotes the average rate at which traffic is transmitted from node u to node v when using i antennas. The network flow must satisfy flow conservation at all nodes when $\lambda l(u)$ flow is sourced (transmitted) from every node $u \in V$. (2) a *feasible* schedule S that decides the set of simultaneous communications (e, i) at time slot t, for $t = 1, 2, \ldots, T$ where T is the period of the schedule and that achieves the network flows in (1). A schedule is feasible if in every time slot the number of antennas dedicated at every node u for communication or for cancelling out interference do not exceed $I(u)$. A schedule achieves a particular flow $f(e, i)$ if the average rate for the communication (e, i) over the T slots is $f(e, i)$.

We denote the problem defined above as a Cross Layer Optimization for MIMO networks problem or CLOM for short. Our main result is an efficient approximation algorithm for the problem.

The algorithm performs the following three steps in the given order:

1) **Solve LP:** We first solve a Linear Programming (LP) based relaxation of the problem optimally. This results in a flow which may not necessarily be realizable using a feasible link communication schedule. However, this flow is "optimal" in terms of ensuring that no better rate can be achieved in the given network. Specifically, this step yields an upper bound on the λ value.
2) **Network transformation:** In this step, we apply a network transformation that adjusts the set of communication links in the network as part of a LP "rounding" process.
3) **Communication Link Scheduling:** In this step the algorithm finds a feasible link communication schedule (over the T time slots) for the transformed network (in previous step) that realizes a scaled version of the flow obtained in Step 1.

In subsequent sections, we describe these steps in detail.

IV. Linear Programming Relaxation

The nodes of the network $G = (V, E)$ can be partitioned into the set of gateway and non-gateway nodes. The gateway nodes provide connectivity to the internet and hence have an aggregate zero traffic demand to be routed through the wireless mesh network.

We define a directed flow graph as follows. The set of nodes in the flow graph are $V \cup \{s, t\}$ for a dummy source node s and a dummy sink node t. The former represents the wireless users that are the traffic source and the latter represent the internet. There is an edge (s, u) for every non-gateway node u of aggregate demand $l(u) > 0$. There is an edge (v, t) for every gateway node v. These edges to and from the dummy nodes have very large capacities to be able to carry any amount of flow in the network. For every feasible communication (e, i) in G, where $e = (u, v) \in E$ there is a directed edge (u, v) in the flow graph. Note that there are at most $\min\{I(u), I(v)\}$

such directed edges (e,i) from node u to node v, namely $(e,1),(e,2),\ldots$. Thus there are multiple parallel edges in the flow graph. The capacity of these pair of directed edges for the communication (e,i) is $c(e,i)$. We denote this flow graph by $F=(V_F,E_F)$. Note that E_F is a multi-set of parallel directed edges, each of which can be identified with a unique feasible communication in G. ¿From now on we will also use the set E_F to also denote the set of all possible feasible communications (e,i) in G. The distinction between E_F representing directed edges or directed communications will be clear from the context. As defined before for G we let $\delta(u)$ denote all the edges in E_F that are incident (incoming or outgoing) on node $u \in V_F$. Among these the incoming edges are denoted by $\delta^-(u)$ and the outgoing edges are denoted by $\delta^+(u)$.

We now define a flow over this flow graph. The flow represents traffic from wireless users to the internet. For an edge $(e,i) \in E_F$, where $e=(u,v)$, the flow represents a communication (e,i) in the direction u to v. We denote this flow by $f(e,i)$ or by $f(u,v,i)$, with the distinction made clear by the context. For this edge let $X_{(e,i)}(t)$ be 1 if there is communication (e,i) from u to v at time slot t and be 0 otherwise. Thus the flow (average rate) $f(e,i)$ for edge $(e,i) \in E_F$ for this communication over T time slots is given by $f(e,i)=\frac{1}{T}\sum_{t\in\{1,\ldots,T\}} X_{(e,i)}c(e,i)$. By re-arranging terms we also get

$$\frac{1}{T}\sum_{t\in\{1,\ldots,T\}} X_{(e,i)} = \frac{f(e,i)}{c(e,i)} \tag{1}$$

We continue to denote the set of wireless nodes by $V=V_F-\{s,t\}$. We formulate the following linear program for a network flow over this flow graph.

$$\begin{aligned}
&\text{maximize } \lambda\\
&\forall v\in V: \quad f((s,v))=\lambda l(v)\\
&\forall v\in V: \quad \sum_{(u,v,i)\in E_F} f(u,v,i) = \sum_{(v,u,i)\in E_F} f(v,u,i)\\
&\forall (e,i): \quad f(e,i)\le c(e,i)\\
&\forall u\in V: \quad \sum_{(e,i)\in E_F, e\in\delta(u)} \frac{f(e,i)}{c(e,i)} \le 1\\
&\forall u\in V: \quad \sum_{(e,i)\in E_F, e\in I_{\delta^-}(u)} \frac{f(e,i)}{c(e,i)} N(e,i) \le KC_q\\
&\forall u\in V: \quad \sum_{(e,i)\in E_F, e\in I_{\delta^+}(u)} \frac{f(e,i)}{c(e,i)} N(e,i) \le KC_q
\end{aligned} \tag{2}$$

We seek to maximize λ (the objective function of the LP) where at least $\lambda l(u)$ amount of throughput can be routed from each non-gateway node u to the Internet (represented by the first constraint). The second constraint is the flow conservation constraint: At any node v which is not the source node s or the sink node t the total incoming flow must equal the total outgoing flow. The third constraint ensures that no capacities are violated.

The fourth constraint is the **Send Receive Constraint:** Recall that the physical model dis-allows a node from engaging in more than one communication (send or receive) at the same time. Thus $\sum_{(e,i)\in E_F, e\in\delta(u))} X_{(e,i)}(t)\le 1$. Averaging over all time slots we get: $\frac{1}{T}\sum_{t\in\{1,\ldots,T\}}\sum_{(e,i)\in E_F,e\in\delta(u)} X_{(e,i)}(t) \le 1$. Thus, from (1) we get this constraint.

The last two constraints are the **Interference Constraint:** We denote by $I_{\delta^-}(u)$ the set of links that interfere with links directed at node u. Thus, $I_{\delta^-}(u)=\cup_{e=(v,u)\in E} I(e)$. Note that these set of links can be thought of as causing interference to transmission received at node u. We assume that the set $\delta^-(u)$ is included in the set $I_{\delta^-}(u)$. Likewise, we denote by $I_{\delta^+}(u)$ the set of links that get interference from links directed out of node u. Thus, $I_{\delta^+}(u)=\cup_{e=(u,v)\in E} I(e)$. Note that these set of links can be thought of as getting interference from transmissions initiated by node u. We assume that the set $\delta^+(u)$ is included in the set $I_{\delta^+}(u)$.

Recall also that node u must dedicate enough DOF for interference suppressions from all simultaneous interfering communications. In addition other nodes must dedicate enough antennas for interference suppressions from transmissions on node u. We model this as the following set of necessary conditions that must be satisfied by any feasible solution. For any node u and any time slot t: $\sum_{(e,i)\in E_F,e\in I_{\delta^-}(u)} X_{(e,i)}(t)N(e,i) \le KC_q$. Also, $\sum_{(e,i)\in E_F,e\in I_{\delta^+}(u)} X_{(e,i)}(t)N(e,i) \le KC_q$. We can show that the term on the right hand side (RHS) of these inequalities (KC_q for some constant C_q dependent only on the ratio q of the interference and transmission ranges) represents a necessary condition (every feasible solution must satisfy these inequalities).

LP upper bound and rounding: It is clear that any optimal solution must satisfy the constraints of the LP and hence by solving this LP, we get a solution which is at least as good as the optimal solution in terms of the minimum fraction of demands (λ) it is able to route. However, the solution may not be feasible in terms of schedulability. In the next section we will show an algorithm that schedules a scaled down (in terms of flows) version of this solution, thus resulting in a feasible solution for the overall problem. As an example, one can easily verify that the solution to the LP of the network in Figure 2 will output $\lambda=3.2$ which is not feasible.

V. Algorithm for Cross Layer Optimization

In this section we design an algorithm that takes as input the flow solution obtained by solving the LP (2) defined in Section IV and turns it into a feasible solution for the CLOM problem. Thus the solution output by the algorithm not just satisfies the constraints for (2) but is also schedulable. Specifically, the output of the algorithm also includes a feasible link communication schedule. λ.

A. Algorithm

We will denote the optimal λ value (as computed by solving (2) in Section IV) as λ^*. The algorithm starts out by finding the smallest number M such that $M\frac{f(e,i)}{c(e,i)}$ is integral for every edge $(e,i) \in E_F$. Here, $f(e,i)$ is the flow on edge (e,i) obtained by optimally solving 2. We would like to note that the integrality requirement for the $M\frac{f(e,i)}{c(e,i)}$ can be relaxed. A smaller value of M for which there is little rounding error introduced by rounding up $M\frac{f(e,i)}{c(e,i)}$ to the nearest integer is sufficient for our purpose. For instance, it can be shown that if the rounding error is at most 50% then the performance of the algorithm is no worse than 50% of when there is no rounding error ($M\frac{f(e,i)}{c(e,i)}$ is integral). Also it can be shown that if M is selected to be much larger than the number of edges in E_F then the impact of the rounding error on the performance of the algorithm is in-significant. We leave the details of these claims for the full paper and for ease of presentation we assume from now on that the choice of M makes all $M\frac{f(e,i)}{c(e,i)}$ integral.

Having found the right M the algorithm then replaces every edge $(e,i) \in E_F$ by $M\frac{f(e,i)}{c(e,i)}$ parallel edges. Let the new network be denoted by $F' = (V_F, E'_F)$. The goal of the algorithm now is to find a way to schedule all the edges in E'_F using a minimum number of time slots. It will be shown later that M is a lower bound on the number of slots needed by even an optimal algorithm. Our algorithm is designed to use no more than a constant multiple of M (constant depends on K) number of slots thus guaranteeing a worst case performance bound.

We now describe the algorithm for scheduling the edges of E'_F. Let the edges $(e,i) \in E'_F$ be ranked in non increasing order of the number of antennas or the value $N(e,i)$ and let $e_1, e_2, \ldots, e_m$ denote this ranking. The algorithm considers the edges in this order and greedily assigns an edge e_i to the first time slot t such that all the edges within time slot t can be involved in simultaneous communications. This in effect means that the following conditions continue to be met for time slot t after adding edge e_i to it:

- **Send Receive Condition:** For each node u at most one edge $(e,i) \in E'_F, e \in \delta(u)$ is assigned to time slot t.
- **Node Antenna Condition:** For any node u the set of antennas dedicated for communication and for interference cancellations in time slot t do not exceed $I(u)$.

Let T_B be the number of slots with at least one edge assigned to them. The algorithm then outputs the schedule corresponding to the assignment of edges to the first T_B slots. We denote this schedule by $E_B(t), 1 \le t \le T$, where $E_B(t)$ is the set of edges in E'_F assigned to slot t and the overall schedule is periodic with a period of T_B slots. In addition the algorithm outputs $\lambda = \frac{M}{T_B}\lambda^*$.

Algorithm 1: Algorithm

Input: Mesh network $G = (V, E)$, Solve (2) (section IV) to determine
  Flow Graph $F = (V_F, E_F)$, Flows $f(e,i)$
  λ^*
Compute Least M so that $M\frac{f(e,i)}{c(e,i)}$ is integral for all $(e,i) \in E_F$
Replace each edge $(e,i) \in E_F$ by $M\frac{f(e,i)}{c(e,i)}$ copies to get graph $F' = (V_F, E'_F)$
Let $e_1, e_2, \ldots e_m$ be edges in E'_F in non-increasing order of number of antennas
Set edge sets $E_B(t) = \emptyset$ for $t = 1, 2, \ldots$, Set $T_B = 0$
For i in 1 to m
  Find least t such that the edges in $E_B(t) \cup \{e_i\}$ satisfy both Send Receive and Node Antenna conditions
  Set $E_B(t)$ equal to $E_B(t) \cup \{e_i\}$
  Set $T_B = \max\{T_B, t\}$
Output:
  $E_B(t), 1 \le t \le T_B$
  $\lambda = \frac{M}{T_B}\lambda^*$

We can show (proof omitted) that the schedule and the λ value output by the algorithm are feasible.

VI. Conclusion

The increasing demand of wireless LANs without the tether of wireline cables paves the way for adopting mesh network technology in enterprises and cities. Streaming applications like voice, video and gaming pushes the capacity and range of the current wireless LAN products to the limit. As a result, vendors have been offering wireless access points and routers with true MIMO technologies for much higher throughput and range, even before the ratification of the 802.11n standard. The benefit of MIMO links constrained in the physical layer will be limited. The full potential of MIMO links to improve network wide performance can only be realized with higher layer considerations. In this paper, we investigate the extent of throughput improvement on the use of MIMO links in wireless mesh networks. Our rigorous framework incorporates cross-layer considerations in routing, scheduling and physical layer MIMO mode selection.

References

[1] Chaska wireless solutions. `http://www.chaska.net/`.

[2] Medford, oregon gets city-wide mobile mesh network. `http://www.muniwireless.com/archives/000258.html`.

[3] M. F. Demirkol and M. A. Ingram. Stream control in networks with interfering mimo links. In *Proc. IEEE WCNC*, pages 343–348, 2003.

[4] S. Gaur, J.-S. Jiang, M. A. Ingram, and M. F. Demirkol. Interfering mimo links with stream control and optimal antenna selection. In *Proc. IEEE GLOBECOM*, 2004.

[5] SOHOware Inc. Aeroguard agn1200 series mimo aps. `http://www.sohoware.com/span/new_sohoware/newfiles/subpages/product/agn%1200.htm`.

[6] K. Sundaresan and R. Sivakumar. Routing in ad-hoc networks with mimo links. In *Proc. IEEE ICNP*, 2005.

[7] K. Sundaresan, R. Sivakumar, M. A. Ingram, and T.-Y. Chang. A fair medium access control protocol for ad-hoc networks with mimo links. In *Proc. IEEE INFOCOM*, 2004.