A Simple Obfuscation Scheme for Pattern-Matching with Wildcards

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Obfuscation

```
void four1(double* data, unsigned long nn)
{
    unsigned long n, mmax, m, j, istep, i;
    double wtemp, wr, wpr, wpi, wi, theta;
    double tempr, tempi;

    // reverse-binary reindexing
    n = nn<<1;
    j=1;
    for (i=1; i<n; i+=2) {
        if (j>i) {
            swap(data[j-1], data[i-1]);
            swap(data[j], data[i]);
        }
        m = nn;
        while (m>=2 && j>m) {
            j -= m;
            m >>= 1;
        }
        j += m;
    };

    // here begins the Danielson-Lanczos section
    mmax=2;
    while (n>mmax) {
        istep = mmax<<1;
        theta = -(2*M_PI/mmax);
        wtemp = sin(0.5*theta);
        wpr = -2.0*wtemp*wtemp;
        wpi = sin(theta);
        wr = 1.0;
        wi = 0.0;
        for (m=1; m < mmax; m += 2) {
            for (i=m; i <= n; i += istep) {
                j=i+mmax;
                tempr = wr*data[j-1] - wi*data[j];
                tempi = wr * data[j] + wi*data[j-1];

                data[j-1] = data[i-1] - tempr;
                data[j] = data[i] - tempi;
                data[i-1] += tempr;
                data[i] += tempi;
            }
            wtemp=wr;
            wr += wr*wpr - wi*wpi;
            wi += wi*wpr + wtemp*wpi;
        }
        mmax=istep;
    }
}
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- Proprietary algorithm?
- Cryptographic keys?

Obfuscation

```c
#include <stdio.h>

            k;double sin()
         ,cos();main(){float A=
       0,B=0,i,j,z[1760];char b[
     1760];printf("\x1b[2J");for(;;
  ){memset(b,32,1760);memset(z,0,7040)
  ;for(j=0;6.28>j;j+=0.07)for(i=0;6.28
 >i;i+=0.02){float c=sin(i),d=cos(j),e=
 sin(A),f=sin(j),g=cos(A),h=d+2,D=1/(c*
 h*e+f*g+5),l=cos      (i),m=cos(B),n=s\
in(B),t=c*h*g-f*        e;int x=40+30*D*
(l*h*m-t*n),y=            12+15*D*(l*h*n
+t*m),o=x+80*y,          N=8*((f*e-c*d*g
 )*m-c*d*e-f*g-l        *d*n);if(22>y&&
 y>0&&x>0&&80>x&&D>z[o]){z[o]=D;;;b[o]=
 ".,-~:;=!*#$@"[N>0?N:0];}}/*#****!!-*/
  printf("\x1b[H");for(k=0;1761>k;k++)
   putchar(k%80?b[k]:10);A+=0.04;B+=
     0.02;}}/*****####*******!!=;:~
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in(B),t=c*h*g-f*        e;int x=40+30*D*
(l*h*m-t*n),y=            12+15*D*(l*h*n
+t*m),o=x+80*y,          N=8*((f*e-c*d*g
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 y>0&&x>0&&80>x&&D>z[o]){z[o]=D;;;b[o]=
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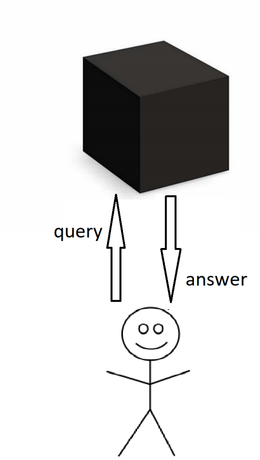

Virtual black-box obfuscation

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Prior work

- Impossible for general circuits [BGI+01]
- Possible for limited function classes such as point functions [LPS04, Wee05] or hyperplane membership [CRV10]
- Most followup work has focused on weaker notions of obfuscation for general circuits following the construction of [GGH+13]

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Our work

- Consider a nontrivial extension and useful to point functions
- Construct *distributional* VBB from a simple assumption

Pattern matching with wildcards

A *pattern* σ is an element $\sigma \in \{0, 1, *\}^n$
$f_\sigma(x) = 1$ if for every bit i, one of the following is true:

- $\sigma_i = x_i$
- $\sigma_i = *$

$w :=$ number of $*$'s can be a constant fraction of n

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Example

$\sigma = 01 * *01$

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Applications

- Non wildcard slots in σ represent a security flaw in code. Want to check for the presence of this flaw without revealing it
- σ matches a problematic input. Want to filter out these inputs without making a user aware if he/she is otherwise unaffected

Pattern matching with wildcards

Prior work

- This function was previously studied by [BR13, BVWW16]
- From multilinear maps and from entropic LWE

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Our wok

- Proof of security in the generic group model
- Simple construction which relies only on elementary algebra to describe and implement

Distributional VBB for pattern matching with wildcards

Distributional VBB security

For every adversary $\mathcal{A}$ there exists a simulator S such that for every distribution $D \in \mathcal{D}_n$ and every predicate $P : \mathcal{C}_n \to \{0,1\}$:

$$\left| \Pr_{C \leftarrow \mathcal{D}_n, \mathcal{G}, \mathcal{O}^{\mathcal{G}}, \mathcal{A}}[\mathcal{A}^{\mathcal{G}}(\mathcal{O}^{\mathcal{G}}(f_\sigma, 1^n)) = P(C)] - \Pr_{C \leftarrow \mathcal{D}_n, \mathcal{S}}[\mathcal{S}^{C}(1^n) = P(C)] \right| = negl(n)$$

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$\mathcal{O}(f_\sigma)$ where $\sigma \sim \mathcal{D}$

- Sample a random pattern σ
- Release obfuscation of f_σ

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Simulator S

- Build 0-function simulator E
- Run $\mathcal{A}$ on E

Generic group model

Setup

- $n \times 2$ table of $2n$ "handles" in $\mathcal{H}$, where h_{ij} corresponds to $x_i = j$

	x_0	x_1	x_2	$\cdots$	x_{n-1}
0	h_{00}	h_{10}	h_{20}	$\cdots$	$h_{(n-1)0}$
1	h_{01}	h_{11}	h_{21}	$\cdots$	$h_{(n-1)1}$

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Group oracle

- Constructs a map $\Phi : \mathcal{G} \to \mathcal{H}$
- Given $h_1, h_2 \in \mathrm{Im}\Phi$, compute $\Phi(\Phi^{-1}(h_1), \Phi^{-1}(h_2))$

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Proper evaluation

- Choose $h_{0x_0}, \cdots, h_{(n-1)x_{n-1}}$ and do some math using group oracle

Proper evaluation

Handle symmetry

Given the pattern $\sigma = 01*$, the following need to behave identically:

x=010	x_0	x_1	x_2
0	$\boldsymbol{h_{00}}$	h_{10}	$\boldsymbol{h_{20}}$
1	h_{01}	$\boldsymbol{h_{11}}$	h_{21}

x=011	x_0	x_1	x_2
0	$\boldsymbol{h_{00}}$	h_{10}	h_{20}
1	h_{01}	$\boldsymbol{h_{11}}$	$\boldsymbol{h_{21}}$

Polynomial interpolation

Setup

- Sample and fix a degree-n polynomial $p \in \mathbb{Z}_p[x]$ such that $p(0) = 0$
- $a_1, \cdots, a_n \sim \mathbb{Z}_p$ and $f(x) = a_1 x + \cdots + a_n x^n$

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Handle distribution

- $\sigma_i \neq j : \tilde{h}_{ij}$ is random in $\mathbb{Z}_p$

Example for $\sigma = 01*$

	x_0	x_1	x_2
0		r	
1	r		

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- $\sigma_i = j : \tilde{h}_{ij} = p(2i + j)$

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	x_0	x_1	x_2
0	$p(0)$	r	$p(4)$
1	r	$p(3)$	$p(5)$

Function evaluation

Function evaluation

- Pick the samples $\{\tilde{h}_{ix_i}\}_{i=0}^{n-1}$
- Constructing interpolating polynomial $\hat{p}$
- Output 1 if $\hat{p}(0) = 0$

Attacks in the clear

Error-correction for Reed-Solomon codes

- Treat the table of $2n$ handles as $2n$ samples of a degree-n polynomial with some number of errors $e = n - w$
- Berlekamp-Welch algorithm can decode if $w > \frac{n}{2}$

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Observations

- Attacks require nonlinear computations over input-output pairs
- Correct evaluation of $\hat{p}(0)$ only requires a linear computation

Construction (in the exponent)

Setup

- Sample and fix a degree-n polynomial $p \in \mathbb{Z}_p[x]$ such that $p(0) = 0$
- Fix a cyclic group $\mathcal{G}$ with generator g and prime order p

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Polynomial interpolation in the exponent

Function evaluation

- $p(x) = \sum_{i=0}^{n-1} y_i b_i(x)$: Lagrange interpolating polynomial over $\{(x_i, y_i)\}$

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Correctness

- If each $h_{ix_i} = g^{p(2i+x_i)}$, then $\prod_{i=0}^{n-1} h_{ix_i}^{C_i} = g^{\sum_{i=1}^{n} p(2i+x_i)C_i} = g^{p(0)}$
- If any h_{ix_i} is a random group element, then output is random

Generic group simulators

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Internal group representation

- S: $\mathcal{G}$

Example element

- $g^{p(3)}$

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- E: $(\mathbb{Z}_p[\mathbf{c}_1, \cdots, \mathbf{c}_{2n}], +)$

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- M: $(\mathbb{Z}_p[\mathbf{a}_1, \cdots, \mathbf{a}_n, \mathbf{b}_1, \cdots, \mathbf{b}_{n-w}], +)$

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- $3\mathbf{a}_1 + 9\mathbf{a}_2$

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- $\mathbf{b}_1$

Security game

Things to keep track of in generic group model

- Correspondence between handles and internal group elements
- When two different generic group simulators differ

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Definition (Simultaneous oracle game)

An adversary is given access to a pair of oracles $(\mathcal{G}_M, \mathcal{G}_*)$, where $\mathcal{G}_*$ is $\mathcal{G}_M$ with probability $1/2$ and $\mathcal{G}_S$ with probability $1/2$. In each round, the adversary asks the same query to both oracles. The adversary wins the game if he guesses correctly the identity of $\mathcal{G}_*$.

Simultaneous oracle game between S and M

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Definition (Evaluation map in the exponent)

Given fixed values $a_1, \cdots, a_n, b_1, \cdots, b_{n-w}$, we have the evaluation map

$$\begin{aligned} \phi : \mathbb{Z}[\mathbf{a}_1, \cdots, \mathbf{a}_n, \mathbf{b}_1, \cdots, \mathbf{b}_{n-w}] &\longrightarrow \mathcal{G} \\ F(\mathbf{a}_1, \cdots, \mathbf{a}_n, \mathbf{b}_1, \cdots, \mathbf{b}_{n-w}) &\longmapsto g^{F(a_1, \cdots, a_n, b_1, \cdots, b_{n-w})} \end{aligned}$$

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- $\mathcal{H}_S^t, \mathcal{H}_M^t$ — the set of handles returned by the simulator up to round t
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- $\Phi_M : \mathbb{Z}[\mathbf{a}, \mathbf{b}] \to \mathcal{H}_M, \Phi_S : \mathcal{G} \to \mathcal{H}_S$ — each simulator's internal mapping of group elements to handles

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2. For every $h^s \in \mathcal{H}_S^t$, $\exists! f \in \mathbb{Z}_p[\mathbf{a}, \mathbf{b}]$ such that $\Phi_S \circ \phi(f) = i_S(h^s)$ and $\Psi^{-1}(h^s) = \Phi_M(f)$

Visualization of (2)

$$\begin{tikzcd}
\mathbb{Z}_p[\mathbf{a},\mathbf{b}] \arrow[r, "\Phi_M{,}\simeq"] \arrow[d, two heads, "\phi"'] & \mathrm{Im}(\Phi_M) & \mathcal{H}_M^t \arrow[l, hook', "i_M"'] \arrow[d, "\Psi{,}="] \\
G \arrow[r, "\Phi_S{,}\simeq"'] & \mathrm{Im}(\Phi_S) & \mathcal{H}_S^t \arrow[l, hook', "i_S"] \arrow[ull, "\exists!"']
\end{tikzcd}$$

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- This is just a combinatorial probability calculation

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Thanks for listening!

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