Optimal neural tuning for arbitrary stimulus priors

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What controls and determines the characteristic shape of a neuron's tuning curve? Previous work has suggested that neural tuning curves are the result of an optimal representation of stimulus information, and thus *e.g.* should change with varying stimulus presentation times [1]. Here, we address the question how such optimal neural tuning should depend on the stimulus distribution.

We first consider the case in which a neuron's firing rates are Poisson distributed with mean firing rate determined by its tuning curve $h(s)$, with constraints $h_{\min} \leq h(s) \leq h_{\max}$. The expected squared reconstruction error of the stimulus magnitude s is bounded by Fisher Information, via the Cramer-Rao bound. This bound can be minimized by the analytic solution of Euler-Lagrange equation. For arbitrary stimulus distribution $p(s)$, the optimal tuning curve is $h(s) = \left[A + B\int_{-\infty}^{s}\sqrt[3]{p(t)}dt\right]^2$. The result reduces to the well-known quadratic form of the optimal tuning curve in the special case of a uniform prior distribution $p(s)$ [2]. We have generalized our analytic derivation to account for other types of noise, such as *e.g.* stimulus dependent, additive Gaussian noise. Furthermore, our analysis can be extended to optimize other information theoretic quantities such as mutual information.

Numerical simulations successfully validated our theoretical results. In addition, we analyzed electrophysiological recordings of the spiking activity of the blowfly H1 neurons , with visual motion stimulus (de Ruyter etal 1997)[3]. By fitting the the observed spiking activity to optimal tuning curve according to our model, we were able to derive the stimulus distribution for which the blowfly neuron is optimally tuned. Our results (Fig. 1 below) suggest that stimulus speed is approximately Gaussian distributed with mean $\mu = 0$ deg/sec and standard deviation $\sigma = 8$ deg/sec. This prediction is in agreement with previous studies that have found a prior for slow stimulus speed [4].

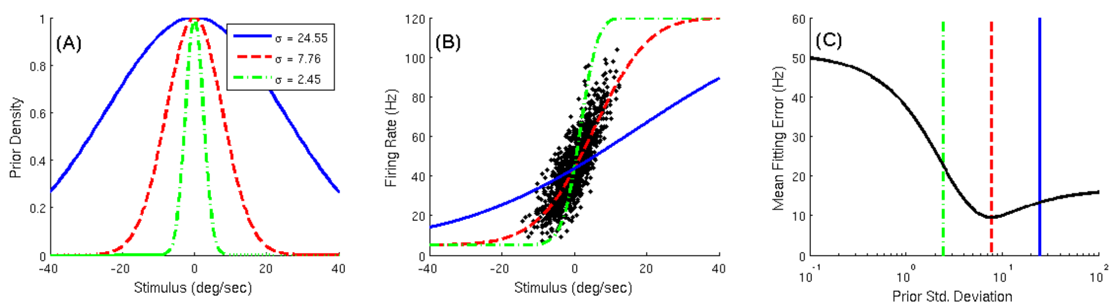

Figure 1: **(A)** Three Gaussian prior distributions $p(s)$ with different values of σ, rescaled to same peak value; **(B)** The theoretical optimal tuning curves for the three priors (corresponding colors and line types). The black dots represent the actual data; **(C)** Mean reconstruction error for different values of σ.

References

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