Supplement to Blended Cured Quasi-Newton for Geometry Optimization

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1 Equivalence

Theorem 1. *For our energy densities $W(\sigma) = f(\sigma)/g(\sigma)$ with $f(\sigma) > 0$ and $g(\sigma) \to 0$ as $\sigma \to 0$, x^* is a stationary point of $\{E(x) : a(x) \geq 0\}$ iff it is a locally injective stationary point of the unconstrained energy $E(x)$.*

Proof. The $1/g(\sigma)$ term drives element energies $W\big(F_t(x)\big) \to \infty$ as $a_t(x) \to 0$. Stationary points x_u^* of unconstrained E are given by $\nabla E(x_u^*) = 0$ and must satisfy $|a(x_u^*)| > 0$. The addition of local injectivity then requires $a(x_u^*) > 0$. Stationary points x_c^* of $\{E(x) : a(x) \geq 0\}$ are given by the Karush-Kuhn-Tucker (KKT) conditions

$$\nabla E(x_c^*) - \nabla a(x_c^*)\lambda = 0 \text{ and } 0 \leq \lambda \perp a(x_c^*) \geq 0. \quad (1)$$

(Here $\lambda = (\lambda_1, .., \lambda_m)^T \in R^m$ is a Lagrange multiplier and x $\perp$ y is the *complementarity condition* $x_t y_t = 0, \forall t$.) All x_u^* satisfy (1) with $\lambda > 0$. For x_c^* satisfying (1) any $\lambda_t = 0 \implies a_t(x_c^*) = 0 \implies W\big(F_t(x_c^*)\big) = \infty$. Thus we must have $\lambda > 0 \implies a(x_c^*) > 0$ so that x_c^* are locally injective stationary points of the unconstrained energy $E(x)$. $\square$

Vertices	Triangles	Iteration CM	Iteration PN	Iteration AKVF
1.9K	3.1K	19	19	23
3.5K	6.3K	19	17	37
6.6K	12.5K	19	27	24
12.9K	25.0K	19	26	24
25.4K	50.0K	20	19	33
50.4K	100.0K	20	25	23
100.4K	200.0K	20	19	38
197.9K	394.6K	20	28	25
435.5K	869.2K	20	29	34
880.3K	1,758.1K	21	36	32
1,650.4K	3,297.5K	21	27	40
3,221.7K	6,438.7K	21	44	36
6,386.2K	12,765.6K	*	*	*
11,969.0K	23,928.4K	*	*	*

Figure 1: AKAP comparison to Newton-type methods. *Here we compare the convergence performance of the AKAP preconditioner to CM and PN in our UV Parameterization Scaling example (Figure 12 in our main paper). We use * to indicate out-of-memory failure for matrix factorization.*

	Vertices	Triangles	BCQN		CM		AKVF		SLIM	
			Iteration	Timing(s)	Iteration	Timing(s)	Iteration	Ratio over CM	Iteration	Ratio over CM
Bull	17.9K	34.5K	169	11.82	25	15.51	25	1.00	148	5.92
Camel	40.2K	78.1K	412	92.60	61	103.32	67	1.10	177	2.90
Dino	24.6K	47.9K	162	30.43	40	31.40	41	1.03	357	8.93
Isis	188.1K	374.3K	347	404.63	33	219.33	37	1.12	†	†
Cow	3.1K	5.8K	78	1.14	27	1.80	25	0.93	143	5.30
Horse	20.6K	39.6K	104	12.93	42	24.38	44	1.05	41	0.98

Figure 2: AKAP and SLIM comparison. *Here we compare the convergence performance of SLIM and the AKAP preconditioner in our UV Parameterization test example with the ISO energy (Figure 15 in our main paper). SLIM and AKAP stencils, and so their fill-in, match CM's (see Figure 8 in the main paper) and thus require the same per-iteration compute cost and storage for linear solutions as CM. Here we report the number of raw iterations as well as the ratio of iterates per method over CM. We use † to indicate when SLIM does not converge.*