Faster Joins, Self-Joins and Multi-Way Joins Using Join Indices

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Abstract

We propose a new algorithm, called Stripe-join, for performing a join given a join index. Stripe-join is inspired by an algorithm called "Jive-join" developed by Li and Ross. Stripe-join makes a single sequential pass through each input relation, in addition to one pass through the join index and two passes through a set of temporary files that contain tuple identifiers but no input tuples. Stripe-join performs this efficiently even when the input relations are much larger than main memory, as long as the number of blocks in main memory is of the order of the square root of the number of blocks in the participating relations. Stripe-join is particularly efficient for self-joins. To our knowledge, Stripe-join is the first algorithm that, given a join index and a relation significantly larger than main memory, can perform a self-join with just a single pass over the input relation and without storing input tuples in intermediate files. Almost all the I/O is sequential, thus minimizing the impact of seek and rotational latency. The algorithm is resistant to data skew. It can also join multiple relations while still making only a single pass over each input relation. Using a detailed cost model, Stripe-join is analyzed and compared with competing algorithms. For large input relations, Stripe-join performs significantly better than Valduriez's algorithm and hash join algorithms. We demonstrate circumstances under which Stripe-join performs significantly better than Jive-join. Unlike Jive-join, Stripe-join makes no assumptions about the order of the join index.

Categories and Subject Descriptors: H.2.4 [Database Management]: Systems - query processing

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1 Introduction

Decision support systems promise significant added value to an organization's information resources. The basic challenge today is to develop efficient query processing techniques that can turn this promise into reality for a wide variety of potential users. The *join* operation of the relational database model is the fundamental operation that allows information from different relations to be combined. Joins are typically expensive operations, particularly when the relations involved are substantially larger than main memory. Therefore, it is critical to implement joins in the most efficient way possible. A number of techniques have been developed to perform "ad-hoc" joins, i.e., joins performed without the benefit of additional data structures such as indices [3, 4, 6, 9, 13, 16].

In this paper, we consider joins of relations for which there exists a precomputed access structure, namely a join index [17]. A join index between two relations maintains pairs of identifiers of tuples that would match in case of a join. The join index may be maintained by the database system, and updated when tuples are inserted and deleted in the underlying relations. In situations where joins are taken often, the cost of doing this maintenance can be more than offset by the savings achieved in performing the join. A number of commercial decision support systems are rumored to be using join indexes [15].

In [17], Valduriez proposed and analyzed a join algorithm that uses the join index. The most important conclusion of that study was that, under many circumstances, having the join index allows one to compute the join significantly faster than the "best" ad-hoc methods such as Hybrid hash-join [6]. However, it was shown in [12] that Valduriez's algorithm utilizes a significant amount of repetitious I/O. Blocks are accessed often for only a small fraction of their tuples. The same block may be read multiple times on different passes within the algorithm.

In [12], Li and Ross proposed two algorithms that significantly improve upon Valduriez's algorithm. The algorithms are called "Jive-join" and "Slam-join." The two algorithms are duals of one another, and have very similar performance. Jive-join range-partitions the tuple-ids of the second input relation, then processes each partition separately. Slam-join forms sorted runs of tuple-ids from the second input relation, then merges those runs.

The crucial virtue of both algorithms is that they make just *a single read pass* through each input relation under lenient memory requirements. Unlike hash-join algorithms, input tuples are not stored in intermediate files; intermediate files contain just short tuple identifiers. Other important features of those algorithms include: (a) Almost all of the I/O performed is sequential. (b) A block of an input relation is read if and only if it contains a record that participates in the join. (c) Skew does not adversely affect the performance. (d) One can join multiple relations, retaining the single-pass property of the inputs, by using multidimensional data structures.

In this paper, we propose a new algorithm that is similar to the Jive-join algorithm of [12]. Our new algorithm, which we term "Stripe-join,"[1] has several important advantages over Jive-join:

- Jive-join requires a fixed ordering of the join index. If the join index is not stored in such an order, one would have to first sort it before applying Jive-join. There is no assumption made about the physical order of the join index in Stripe-join. Thus, Stripe-join can avoid this sorting step.

- Stripe-join is symmetric with respect to all relations. The join index is first processed, then all participating relations are processed in a symmetric fashion. Jive-join, on the other hand, treats one relation in particular as "special" by reading it at the same time as the join index. The symmetry of Stripe-join has an important performance benefit: For joins in which a relation R appears more than once, Stripe-join can make a single pass through R to cover *all* of the instances of R in the join. In particular, self-joins can be performed with just one pass through the self-joined relation. We believe that the symmetry property also renders Stripe-join easily extensible to parallel architectures.

- Stripe-join delivers better performance under some circumstances, in particular when the cardinality of the join index is high.

Like Jive-join, Stripe-join uses a vertically partitioned data structure for the join result. (We *do not* assume that the *inputs* are vertically partitioned.) Attributes from the first input relation are stored in a separate file from those of the second input relation, using transposed files [1]. Attributes that are common are placed arbitrarily in one of the two vertical fragments. There is a one-to-one correspondence between records in each vertical partition: The nth record in the first vertical fragment matches the nth record in the second. Such a representation has a negligible performance impact on processes that read the join result [12]. It is our understanding that vertical partitioning is being deployed in some current proprietary commercial database systems, including Sybase IQ and Red Brick.

Our main contributions include:

- The proposal of a novel algorithm for joining relations using a join index. To our knowledge this is the first algorithm that, given a join index and a relation significantly larger than main memory, can perform a self-join with just a single pass over the input relation and without storing input tuples in intermediate files.

- The analytic performance analysis of the algorithm using a detailed cost model, demonstrating efficient single-pass performance under lenient memory requirements.

[1] "Stripe" is both an acronym (Symmetric Treatment of Relations that have a join Index and that are Particularly Extensive) and a phrase intended to allude to the vertically partitioned nature of the join result.

- Analytic comparisons of our algorithms with other algorithms, demonstrating significant performance improvements under several conditions.

The structure of the paper is as follows. In Section 2 we discuss our assumptions and present the vertically partitioned data structure for the join result. In Section 3 we present our Stripe-join algorithm, which is then analyzed in Section 4. Section 5 compares our algorithms with other algorithms. Section 6 discusses various extensions of our algorithms. We conclude in Section 7.

2 Background

We consider an r-way join of r relations. (A conventional join has $r = 2$; while the most common type of join is probably a two-way join, the description of our algorithms is only slightly more complex in the r-way case, and so we simply present the r-way join algorithm directly.) The input relations to be joined are denoted by $R_1, \ldots, R_r$. We assume for simplicity that a tuple-id for each input relation is simply the "position" of the tuple within the relation (eg., 1, 2, ...). However, our algorithm applies for any sequential physical addressing scheme for which input relation records in one block have smaller tuple-id values than the records in the next block. Tuple-ids are used in the join index and in the intermediate results of our join algorithm.

A join index is the set of tuples $(t_1, \ldots, t_r)$ of tuple-ids such that the tuple-ids t_i from R_i refer to tuples that match according to the join condition. We shall treat the join index as a relation, and denote it by J. We do not assume that the join index is physically ordered by any attribute.

For simplicity, we shall assume that all of the attributes of each R_i are required in the join result. The extension of our analysis to cases where fewer attributes are required is straightforward.

We do not assume that any indexes are available on the input relations. We also do not assume that either input relation is physically ordered by any attribute.

Following [8], we assume that separate disks are used to store (a) join indices, (b) temporary files, (c) input relations, and (d) the output result. By using separate disks we avoid unnecessary disk seeks between accesses. The input relations may reside on the same disk as each other.[2] Similarly, we can store all temporary files on one disk. (This kind of configuration is recommended by most commercial vendors.)

[2]Actually, for the Stripe-join algorithm one can store the join index on the same disk as the input relations with no loss of performance.

2.1 A Vertically Partitioned Data Structure for the Join Result

We use a vertically partitioned data structure known as a transposed file [1] to store the join result. Attributes from each R_i that are present in the join result are stored in a separate file (denoted JR_i). Join attributes that are common to more than one relation are placed arbitrarily in one of the vertical fragments. The first entry in each of the files corresponds to the first join result tuple, the second entry to the second join result tuple, and so on. There is no need for any additional stored tuple-id or surrogate key. Each vertical fragment is in the same sequence. This layout is summarized in Figure 1 for $r = 2$. The join result JR is shown on the left in a traditional

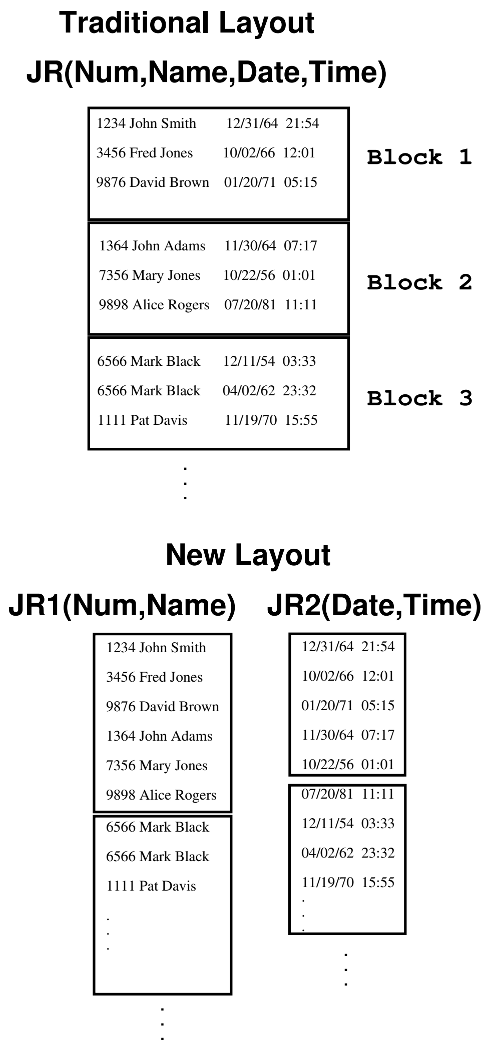


Figure 1: Physical Data Layout for the Join Result.

layout, and on the right in a partitioned layout as JR_1 and JR_2. The input relation R_1 has attributes *Num* and *Name*, and R_2 has attributes *Num*, *Date* and *Time*.

The advantage of the partitioned representation is that we will be able to write each vertical partition in a separate phase of the algorithm, getting better utilization of main memory.

3 Stripe-join

In this section we present a new algorithm called Stripe-join for performing joins using a join index. The algorithm range-partitions the join index into a number of temporary files. The input relations are then processed with their corresponding temporary files to generate the vertical partitions of the join result.

Algorithm 3.1: (Stripe-join) The algorithm consists of three steps:

Step 1

For $i = 1, \ldots, r$ we choose $k_i - 1$ tuple-ids as partitioning elements for R_i. The aim is to evenly partition the tuples participating in the join. We defer the discussion on how the partitioning values are chosen until Section 4.3. Thus there are $y = k_1 k_2 \cdots k_r$ partitions altogether; we number them $1, \ldots, y$ in lexicographic order. We refer to the collection of y/k_i partitions corresponding to a single partitioning range over R_i an *i-segment*.

For each $i = 1, \ldots, r$ we allocate in memory k_i *temporary file buffers* $Z_i^1, \ldots, Z_i^{k_i}$ each of length v blocks.

Step 2

We scan J sequentially. For each join-index tuple $(t_1, \ldots, t_r)$ we first identify the partition to which this tuple belongs, based on the $R_1, \ldots, R_r$ tuple-ids. Suppose it is partition number l (between 1 and y). We also identify the segment to which this tuple belongs within each relation R_i. Thus we have for each $i = 1, \ldots, r$ an i-segment number g_i (between 1 and k_i) corresponding to the position of t_i among the partitioning elements for R_i.

For each $i = 1, \ldots, r$ we write the pair (t_i, l) to $Z_i^{g_i}$. If the buffer $Z_i^{g_i}$ is filled, then it is flushed to disk, after which it can accept new values. When all tuples have been processed in this way, all buffers are flushed to disk, and the memory for the buffers is deallocated.

After finishing Step 2, we have generated $k_1 + \cdots + k_r$ temporary files which we use in Step 3 below.

Step 3

We perform the following steps for each i in $\{1, \ldots, r\}$.

For each of the k_i temporary files (each corresponding to an i-segment) we perform the following operations. We read into memory the whole temporary file, processing each (t_i, l) pair by appending the t_i value to a list corresponding to the l value. We call these lists the *grouped lists*.

Additionally, we compose a second array consisting of all the t_i values from the temporary file, sorted in tuple-id order with duplicates removed. We then retrieve tuples from R_i in order, retrieving only blocks that contain a matching record according to our sorted array of tuple-ids. We stop when we reach the end of the array.

We are now ready to write the segment's portion of JR_i. There are y/k_i partitions in the segment, corresponding to the different values of l in the (t_i, l) pairs. We write the full R_i tuples for each partition to a separate file, in the order of the corresponding grouped list above. (We could use binary search to locate the R_i tuples in order, or alternatively store the R_i tuples in a hash table by hashing on the tuple-id.)

By the time we have finished with the final i-segment, we have generated all of JR_i. The partitions of JR_i can be linked together[3] into a single file. Once we have performed Step 3 for all values of i we have the required join result. □

We illustrate the use of Stripe-join using an example based on [12], but without an ordered join index.

Example 3.1: Consider the two relations *Student* and *Course*, and their join result, given below. This particular join is a natural join in which we match the course numbers in the two input tables. To enable the reader to keep track of various duplicate student tuples as they are processed through the algorithm, we have provided superscripts to help distinguish them.

Student	Course
Smith^1	101
Smith^2	109
Jones	104
Davis^1	102
Davis^2	105
Davis^3	106
Brown	102
Black	103
Frick	107

Relation *Student*

Course	Instructor
101	Green
102	Yellow
103	Green
104	White
105	Evans
106	Alberts
106	Beige
108	Red
109	Grey

Relation *Course*

[3] We can physically place the partitions of JR_i contiguously on disk if we keep track in Step 2 of the number of tuples in each partition. In this case the linking step is trivial.

Student	Course	Instructor
Brown	102	Yellow
Smith^1	101	Green
Jones	104	White
Smith^2	109	Grey
Davis^3	106	Alberts
Davis^1	102	Yellow
Davis^3	106	Beige
Black	103	Green
Davis^2	105	Evans

Join Result

S-id	C-id
7	2
1	1
3	4
2	9
6	6
4	2
6	7
8	3
5	5

Join Index

For the purposes of exposition, we assume that each input record occupies one disk block. We also assume that we have three partitions for each relation ($k_1 = k_2 = 3$, $y = 9$) and that each buffer can hold just one input record at a time. We choose partitioning values 4 and 6 for the tuple-ids of the matching *Student* tuples, and values 3 and 6 for the tuple-ids of the matching *Course* tuples. The partition numbering is summarized in the following diagram:

Student tuple-id	*Course* tuple-id (1) id<3	(2) $3 \leq$id< 6	(3) $6 \leq$id
(1) id< 4	1	4	7
(2) $4 \leq$id< 6	2	5	8
(3) $6 \leq$id	3	6	9

After the first step, we have allocated six buffers, three for each of *Student* and *Course*. After the second step, we have partitioned the join index (and added partition numbers) as follows:

Student

(1)	(2)	(3)
(1,1)	(4,2)	(7,3)
(3,4)	(5,5)	(6,9)
(2,7)		(6,9)
		(8,6)

Course

(1)	(2)	(3)
(2,3)	(4,4)	(9,7)
(1,1)	(3,6)	(6,9)
(2,2)	(5,5)	(7,9)

In Step 3 of the algorithm, for each temporary file, we read in and divide the temporary file sequences into the grouped lists according to the partition number (the second element of the pair). We also generate a sorted list of tuple-ids appearing in the file. The corresponding tuples are then read from the input relation and written to fragments of the join result, one fragment for each partition. This process is summarized in Figure 2.

The left column in Figure 2 contains the grouped list for each partition number. The middle column contains the tuple-ids in sorted order (without duplicates), and the right column contains the matching records from the input relation that are read in. The records are written to the output in the order of the grouped lists. Since we generate one partition at a time, we need

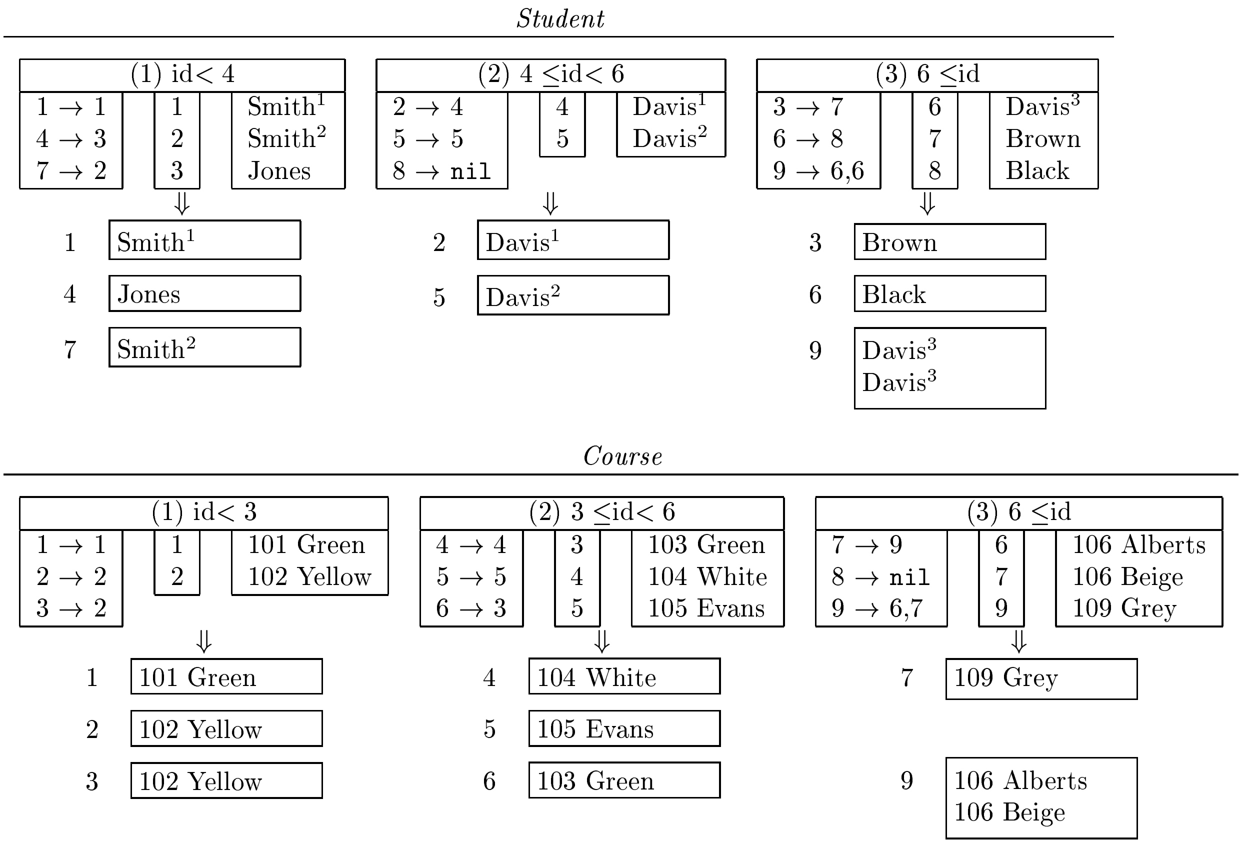


Figure 2: Step 3 of Stripe-join for Example 3.1.

only one output buffer in memory to generate the partitions. Concatenating the various partitions of the generated join result in order of the partition numbers, our output looks like this:

Student
Smith^1
Davis ^{1}
Brown
Jones
Davis^2
Black
Smith^2
Davis^3
Davis^3

Course	Instructor
101	Green
102	Yellow
102	Yellow
104	White
105	Evans
103	Green
109	Grey
106	Alberts
106	Beige

The output is in two vertical fragments JR_1 and JR_2. The horizontal lines denote the boundaries between the partitions. The result is the same as that given initially, except for the order of the tuples. □

An important aspect of Stripe-join is that tuples from the various relations "line up" within each partition. The design of the algorithm ensures that within a partition the tuples are in the same order as they were in the initial join index, ensuring that the tuples do in fact line up.

Stripe-join shares a number of important characteristics with Jive-join, as illustrated by Example 3.1: (a) The algorithm reads only participating records. Records for student Frick and instructor Red are not read from disk, saving I/O. (b) Participating records are read just once, even when they participate multiple times. (c) By using buffering, the algorithm can operate while keeping at most three full records (from either relation) in memory at any one time, together with some tuple-ids and partition numbers. Both input relations are larger than three records. (d) Only one pass over each input relation is made. In contrast, with main memory capable of holding just three records (plus some tuple-ids), Valduriez's algorithm would make three passes through the Course relation. We shall discuss the *improvements* that Stripe-join makes over Jive-join in Section 5.

3.1 An Enhancement

Notice that in Example 3.1 the output result partitions for the *Course* relation are actually generated in the same order that they appear in the join result. One could combine the partitions into a single large unit within Step 3 of the algorithm, thus generating

101	Green
102	Yellow
102	Yellow
104	White
105	Evans
103	Green
109	Grey
106	Alberts
106	Beige

in one unit rather than in eight partitions. As we shall see, reducing the number of separate output partitions will improve the performance by requiring fewer disk seeks.

We can apply this enhancement fully to just one of the participating relations, because the order of the partition numbers can agree with at most one of the input relations' segment orders. However, it is possible to apply it partially to other relations.

For example, consider a three-way join in which each relation is partitioned in two ($k_1 = k_2 = k_3 = 2$). Thus there are eight partitions in total, numbered 1 to 8. For R_1, we may process the partitions as $\{1, 2, 3, 4\}$ in the first 1-segment, followed by $\{5, 6, 7, 8\}$ in the second 1-segment. Thus we can fully utilize the enhancement for this relation. For R_2, we may process the partitions as $\{1, 2, 5, 6\}$ in the first 2-segment, followed by $\{3, 4, 7, 8\}$ in the second 2-segment. We can reduce the effective number of partitions by half by writing partitions 1 and 2 to the same output unit. Similarly, we can write partitions 5 and 6, partitions 3 and 4, and partitions 7 and 8 each together. (It does not significantly help to write 3 and 4 to the same unit of output that 1 and 2 were previously written to because intervening seeks are still necessary.) For R_3 we would process $\{1, 3, 5, 7\}$ and $\{2, 4, 6, 8\}$ in the two segments, with no opportunities for applying the enhancement.

Let us suppose that the R_1 partition order is the highest-order component of the partition order. Then R_1 can reduce the number of partitions to 1; for $i = 2, \ldots, r$, R_i can reduce the number of partitions to $k_1 \cdots k_i$ from y.

3.2 Self-Joins

There is an important specialization of the Stripe-join algorithm for the case where a relation is joined with itself. For an example of a self-join, consider a relation

```
Employee(ssn,name,salary,manager-ssn)
```

and suppose we wish to compare the salaries of employees with the salaries of their managers. In order to do so we join `Employee` with itself, equating `ssn` in one copy with `manager-ssn` in the second.

Suppose we had a join index for this join, i.e., a set of pairs of tuple-ids of employees' records with tuple-ids of their managers' records. Then we could apply Stripe-join as described above. However, that would result in two passes over the `Employee` relation. We can do better by coalescing the two passes over the `Employee` relation in Step 3 of the algorithm into one pass during which time we do the work for both of the copies of `Employee` in the join. This optimization can be used whenever a relation appears more than once in the join.

Algorithm 3.2: (Stripe-join with Repeated Relations)

The algorithm is identical to Algorithm 3.1 except as noted below.

In Step 1 we require any relation that appears more than once in the join to be partitioned in the same way each time it is mentioned. Thus the k_i and k_j values (and the corresponding partitioning elements) are the same if R_i and R_j are actually the same relation.

We modify Step 3 as follows. Suppose that $R_{i_1}, \ldots, R_{i_n}$ is the (maximal) set of instances of the same relation R in the join, with $n > 1$. Let $k = k_{i_1} = \cdots = k_{i_n}$. For $j = 1, \ldots, k$ we proceed as follows:

> We read into memory the jth temporary file from all of $R_{i_1}, \ldots, R_{i_n}$. Thus we have n temporary files, one per relation instance, in memory simultaneously. Each of the temporary files is separately grouped into lists as before. However, we produce only one combined duplicate-free list of tuple-ids for tuples of R that participate in the join. Those tuples of R mentioned in the list are read sequentially into memory. The remainder of this step is as before: we output the various partitions for each of $R_{i_1}, \ldots, R_{i_n}$ while we have the R tuples in memory.

If more than one relation appears multiple times, then we modify the algorithm as above for each repeated relation. □

4 Performance Analysis

4.1 A Detailed Join Cost-Model

A table of symbols is given in Table 1. A table of system constants, with their value (used in the analytic comparisons) is given in Table 2. The constants used follow [8, 5, 12], and correspond to the Fujitsu M2266 disk drive.

Symbol	Meaning
$\vert R \vert$	Number of blocks in R.
$\vert\vert R \vert\vert$	Number of tuples in R.
$\ll R \gg$	Width in bytes of a tuple in R.
r	Number of participating relations.
t	Size in bytes of a tuple-id, in-memory pointer, or integer value.
τ_i	Semijoin selectivity, i.e., the proportion of the tuples in R_i that participate in the join.
$Y(k, d, n)$	Function to estimate the number of block accesses needed to retrieve k tuples out of n tuples stored in d blocks [19].
m	Size of main memory, in disk blocks.
β	Blocks in an input relation buffer.
N_S	Seeks in an algorithm.
$N_{I/O}$	I/O requests in an algorithm.
N_X	Block transfers in an algorithm.

Table 1: Table of symbols

Haas, Carey and Livny have proposed a detailed I/O cost model in which seek time and rotational latency are explicit [8]. These authors reexamine a number of ad-hoc join methods using their cost model, and demonstrate that the ranking of join methods obtained by using a block-transfer-only cost model for I/O may change when the same algorithms are analyzed using their more detailed cost model. In this paper, we shall use the detailed cost model from [8] to measure the cost of various join algorithms. The total I/O cost of a join algorithm is measured as

$$N_S T_S + N_{I/O} T_L + N_X T_X.$$

Symbol	Value	Meaning
b	8192	Bytes in a disk block.
c	83	Disk blocks in a cylinder.
D	130000	Blocks in a disk device.
T_S	9.5	Time for an average disk seek (milliseconds).
T_L	8.3	Average rotational latency (milliseconds).
T_X	2.6	Block transfer time (milliseconds).

Table 2: Table of system constants

In other words, the total I/O cost of an algorithm is the sum of three component costs: seek cost, latency cost, and page transfer cost. Each of these costs is in turn the product of the number of actions multiplied by the average time that action consumes.

In this paper we ignore CPU cost, and focus on the I/O cost. The main reasons for this choice are (a) that CPU cost is significantly smaller than the I/O cost, and (b) almost all of the CPU-intensive work can be done while waiting for disk I/O. Furthermore, I/O cost will be even more dominant in the future, as processor and memory speeds are increasing five to ten times faster than I/O device speed. Due to space constraints, we defer the presentation of our CPU cost model to the full version of this paper. The dominance of I/O cost over CPU cost for Jive-join has been demonstrated experimentally in [12].

We perform input buffering on the input relations in order to reduce seek and rotational latency. If the input buffer size is β blocks, then that is the minimal unit of information transfer from the input relations: we must read a β-block chunk if any of the constituent blocks contains a participating tuple. In our analytic graphs we use a value of β equal to the cylinder size c, i.e., 83 blocks.

Some of our disk output is not fully sequential. We shall allocate buffers and optimize their size to get the best I/O cost.

In some stages of our algorithm, records are accessed in order from a contiguously stored relation. We can approximate the total seek time for one pass through relation R as $3|R|/D$ times the average seek cost, where D is the capacity (in blocks) of the disk unit. We count three times[4] the "average" seek cost, estimating that the average seek cost is equal to one third of the time taken to move from one edge of the disk to the other. This rough approximation assumes that seek time can be accumulated in a linear fashion, and that there are no competing accesses to the disk device. If there

[4]One can show analytically that for linear seek times the time taken to traverse the whole disk is, on average, three times the time taken to move from a random cylinder to another random cylinder on the disk.

was contention on the disk device between cylinder accesses, then we would have to count one seek per cylinder, since the seeks between cylinders would not necessarily be small.

We assume that in-memory sorting is done in-place.

In this paper we do not address the cost of maintaining the join index. Blakeley and Martin have comprehensively analyzed the tradeoff between join index maintenance cost and the join speedup [2].

4.2 Memory Requirements

We need Step 1 and Step 2 to fit in main memory. Ignoring insignificant terms, the following inequality must hold:

$$v(k_1 + \cdots + k_r) \leq m. \tag{1}$$

Each iteration of Step 3 must also fit in main memory. We assume that the tuples in the k_i temporary files for relation R_i are evenly distributed (see Section 4.3). The sorted tuple-id list can be discarded incrementally (and the memory reused) as the R_i records are read. Thus, we get

$$|J|/r + \tau_i|R_i| \leq k_i m \tag{2}$$

assuming that all of the attributes of R_i are required in the join. Combining Equations 1 and 2 with the constraint that $v \geq 1$ yields

$$m \geq \sqrt{|J| + \tau_1|R_1| + \cdots + \tau_r|R_r|}. \tag{3}$$

Equation 3 specifies the minimum amount of memory necessary for Jive-join to perform with a single pass over the input relations. This is a very reasonable condition, stating that the number of blocks in main memory should be at least of the order of the square root of the number of participating blocks in all relations, and the number of blocks in the join index.

To get an idea of how lenient this constraint is in practice, imagine we had 128 megabytes of main memory, that disk blocks were 8K bytes, and that we had a one-to-one two-way join between R_1 and R_2 with full participation by both relations. Assuming that tuples in the input relations are much wider than a tuple-id, we would be able to apply Stripe-join with a single pass through each input for inputs of total size up to 2 terabytes. (For larger relations, Stripe-join still applies, but with higher cost; see Section 6.)

In Section 4.4 we will show how to choose optimal values of k_i and v.

For Self-joins we keep more information in memory during Step 3. Let us consider an r-way self join of a relation R. We can estimate the combined semijoin selectivity over the r uses of R as $\tau = 1 - (1 - \tau_1) \cdots (1 - \tau_r)$ by assuming that the semijoin selectivities are independent. Let $k = k_1 = \cdots = k_r$. Equation 2 now becomes

$$|J| + \tau|R| \leq km \tag{4}$$

since we keep r sets of tuple-ids in memory at once, rather than one. Consequently, Equation 3 becomes

$$m \geq \sqrt{r|J| + \tau|R|}. \tag{5}$$

Equation 5 is an improvement over Equation 3 (with right-hand-side $\sqrt{|J| + (\tau_1 + \cdots + \tau_r)|R|}$ for self-joins) when the $|R|$ term dominates the $|J|$ term.

4.3 Choosing the Partitioning Values

We now show how to choose the partitioning elements in Step 1 of Stripe-join. A first attempt might be to partition the tuple-ids evenly. Since the number of tuples in each R_i is known, we could simply divide the tuple-id range into k_i equal-sized partitions.

This approach would work if the distribution of tuples in the join was uniform. However, for distributions with significant skew, we may find that some partitions contain many more participating tuples than others. For all partitions to fit in main memory, we would have to ensure that the largest partition, together with its portion of the temporary file, fits in main memory in Step 3. We thus waste some memory for all other partitions.

Fortunately, we can do better. In fact, we can *perfectly* partition the tasks of Step 3 to just fit into memory if we are prepared to perform a preprocessing step on the join index. The join index provides us with all the information we need about skew. A preprocessing step like that of [12] can examine the join index and calculate the partitioning elements that divide the tasks of Step 3 into equal-sized chunks.

Another alternative is for the system to *maintain* a set of partitioning values at the same time that it maintains the join index. In a low-update environment such as a decision support system, such an approach would be feasible.

4.4 Measuring the Cost

We now calculate the values of N_S, $N_{I/O}$, and N_X in order to measure the cost of Stripe-join. We do not include the block transfer cost of writing the output because it is the same for all algorithms. We let z denote the total size of the temporary files: $z = |J| + |J|\lceil(\log_2 y)/8\rceil/t$ since we need $\log_2 y$ bits to represent the range $1, \ldots, y$. We assume that the partitioning values have already been chosen and do not need any significant I/O to read in. Thus, there is no measured I/O in Step 1.

The number of seeks in Step 2 is $3|J|/D$ for J, plus one seek for each buffer flush. The number of buffer flushes is z/v. The number of seeks in Step 3 is $3(|R_1| + \cdots + |R_r|)/D$ for $R_1, \ldots, R_r$ (since each R_i is read sequentially), $k_1 + \cdots + k_r + 3z/D$ for reading the temporary files, plus one seek for each time one starts a new partition in the output result. Using the enhancement of Section 3.1, we count the accesses in one dimension (say R_1) as sequential, with total seek cost $3(||J||* \ll R_1 \gg)/Db$; in the other dimensions the total seek cost is $(r-2)y/k_1 + y$. Thus, we obtain the formula

$$\begin{aligned} N_S &= \tfrac{3}{D}\,(|J| + |R_1| + \cdots + |R_r| \\ &\quad + ||J|| * \ll R_r \gg /b + z) \\ &\quad + k_1 + \cdots + k_r + z/v + (r-2)y/k_1 + y. \end{aligned}$$

The number of I/O requests in Step 2 is $|J|/\beta$ for J and z/v for writing the temporary file buffers. The number of I/O requests in Step 3 is $Y(\tau_1||R_1||, |R_1|/\beta, ||R_1||) + \cdots + Y(\tau_r||R_r||, |R_r|/\beta, ||R_r||)$ for reading all of the R_i relations, plus $k_1 + \cdots + k_r$ for reading the temporary files, plus ry for writing the join result. Thus, we obtain the formula

$$\begin{aligned} N_{I/O} &= |J|/\beta + Y(\tau_1||R_1||, |R_1|/\beta, ||R_1||) + \cdots \\ &\quad + Y(\tau_r||R_r||, |R_r|/\beta, ||R_r||) \\ &\quad + k_1 + \cdots + k_r + z/v + ry. \end{aligned}$$

The number of block transfers in Step 2 is $|J|$ for reading J and z for writing the temporary files. In Step 3 we read the temporary files, with cost z. The cost for reading the input relations is $\beta Y(\tau_1||R_1||, |R_1|/\beta, ||R_1||) + \cdots + \beta Y(\tau_r||R_r||, |R_r|/\beta, ||R_r||)$. Thus, we obtain the formula

$$\begin{aligned} N_X &= |J| + 2z + \beta Y(\tau_1||R_1||, |R_1|/\beta, ||R_1||) + \cdots \\ &\quad + \beta Y(\tau_r||R_r||, |R_r|/\beta, ||R_r||). \end{aligned}$$

Now we are in a position to choose optimal values for $k_1, \ldots, k_r$ and v. The algorithm cost is an increasing function of each k_i, and a decreasing function of v. Thus we wish to minimize each k_i and maximize v.

Thus we can interpret Equation 2 as stating that

$$k_i = (|J|/r + \tau_i|R_i|)/m.$$

Equation 1 then implies that

$$v = \frac{m^2}{(|J|/r + \tau_1|R_1|) + \cdots + (|J|/r + \tau_r|R_r|)}.$$

In an analogous fashion we can determine the cost of r-way self-joins using Stripe-join. We obtain

$$\begin{aligned} k &= (|J| + \tau|R|)/m \\ v &= m^2/r(|J| + \tau|R|) \\ N_S &= \tfrac{3}{D}\,(|J| + |R| + ||J|| * \ll R \gg /b + z) \\ &\quad + rk + z/v + (r-2)y/k + y. \\ N_{I/O} &= |J|/\beta + Y(\tau||R||, |R|/\beta, ||R||) \\ &\quad + rk + z/v + ry. \\ N_X &= |J| + 2z + \beta Y(\tau||R||, |R|/\beta, ||R||). \end{aligned}$$

4.5 Selection Conditions

Stripe-join applies when one uses a *subset* of the join index rather than the full join index. Thus one could apply selection conditions to the input relations' indexes (which are much smaller than the input relations themselves), combine the lists of resulting tuple-ids (using union for an "or" condition and intersection for an "and" condition [14]), and use the result to semijoin the join index [17]. Stripe-join can then be applied to the input relations and the reduced join index. Similarly, if the join index also included the join attribute value (for easy maintenance) then selections on the join attribute could be made on the join index itself.

5 Comparing Stripe-join with Other Algorithms

In this section we compare Stripe-join with several other algorithms. We present performance graphs for several example scenarios that illustrate the analytically derived cost for each algorithm.

5.1 Hash Joins

There are several variants of hash joins in the literature. Two important variants are Grace-hash join [10] and Hybrid-hash join [6]. In Grace-hash join the input relations are hashed on the join attribute in such a way that the disk-resident hash buckets of one relation fit in memory. In a second phase the algorithm performs an in-memory join of the records in the corresponding hash-buckets. Hybrid-hash join additionally keeps one hash bucket of the first relation in memory. When the first relation is not much larger than main memory the in-memory hash bucket significantly reduces the amount of I/O. The I/O performance of Hybrid-hash join has been studied in [6, 8, 12]; the graphs in this paper use the formulas from [12].

One can formulate a version of Grace-hash join that applies specifically to self-joins. During an initial pass of the relation to be joined, one can simultaneously hash on *both* attributes that are matched for the join. Thus one can save an entire pass through the input relation, although one still has to write and read the two disk-resident hash tables. We refer to this algorithm as a "Self-hash" join. Its performance characteristics can easily be derived from those of Grace-hash join [10, 8]. The same technique may be incorporated into Hybrid-hash join when performing self-joins, but we shall not consider it further. For input relations significantly larger than main memory, Hybrid-hash delivers essentially the same performance as Grace-hash [8].

5.2 Valduriez's Algorithm

Assuming the join index is clustered on one of the tuple-id fields (say, R1 tuple-id), Valduriez's algorithm joins two input relations as follows [17]. As much of J and R_1 as will fit in memory is read in sequentially from secondary storage. Then the in-memory portion of the join index is sorted by the tuple-id value from R_2 and R_2 is scanned sequentially for the tuples that match the memory-resident J tuples. For each R_2 tuple retrieved, the corresponding R_1 tuple is located in memory, and the resulting join tuple is written to the output file. The above process is repeated until J and R_1 have been exhausted. This algorithm requires that R_2 be scanned multiple times when J and R_1 cannot fit in memory at once. See [12] for the exact cost formula.

5.3 Jive-join

Jive-join improved on previous join techniques by allowing one to perform the join using a single sequential scan of the input relations. Stripe-join was inspired by Jive-join.

Example 5.1: Figure 3 shows a graph (based on one from [12]) that compares Jive-join (and Slam-join), Valduriez's algorithm, Hybrid-hash join and Stripe-join. The join is a two-way one-to-one join between two relations of size 2^{25} ($\approx$ 34 million) tuples of width 256 bytes. All tuples participate in the join. The performance of Stripe-join and Jive-join is very similar. The cost of Jive-join includes the cost of sorting the join index (see Appendix A). As can be seen from the graph, Jive-join and Stripe-join perform close to the lower bound, and significantly better than both Hybrid-hash join and Valduriez's algorithm. □

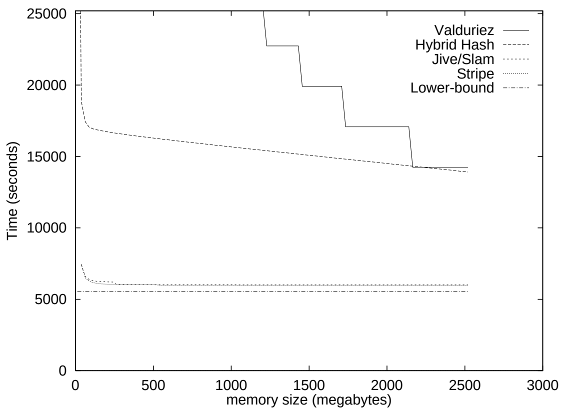


Figure 3: Performance graph for Example 5.1.

For a complete description of Jive-join, and a thorough performance comparison of Jive-join with other algorithms see [12]. In no scenarios that we have studied have we found examples where Jive-join significantly outperforms Stripe-join. In cases like Example 5.1, Stripe-join and Jive-join have comparable performance characteristics. In those cases the comparisons of Jive-join with other techniques apply equally well to Stripe-join, but we do not repeat all of the comparisons from [12] here. Instead, we focus on the cases where Stripe-join can significantly improve upon Jive-join. Jive-join and Slam-join are dual algorithms with very similar characteristics. Thus, our comparison of Stripe-join with Jive-join below also holds for Slam-join.

5.4 A Comparison

When there is sufficient main memory the contribution of seek time and rotational latency is small. In that context, we compare the N_X values for Stripe-join and Jive-join.[5] The saving of Stripe-join over Jive-join is $N_X^{\textit{Stripe-join}} - N_X^{\textit{Jive-join}}$ which is equal to

$$|J|(2 - 2/r + 2\lceil \log_m \lceil |J|/m \rceil \rceil - 2\lceil (\log_2 y)/8 \rceil / t)$$

If we make the reasonable assumptions that $m < |J| \leq m^2$ and that $\lceil (\log_2 y)/8 \rceil = t$, then the saving is equal to $(2 - 2/r)|J|$. In other words, Stripe-join does better (between one and two passes' worth of the join index) than Jive-join when there is both plenty of memory and an unsorted join index. If $|J| \leq m$, so that the join index fits in memory, then the difference becomes $(-2/r)|J|$ meaning that Jive-join is slightly better than Stripe-join (up to one passes' worth of the join index). To illustrate that the improvement of Stripe-join over Jive-join may be significant, consider the following example.

Example 5.2: Suppose we fix the size of two input relations and vary the number of tuples in the join result. More specifically, let R_1 and R_2 both have width 256 bytes, and both have 2^{25} (about 34 million) tuples. Suppose that main memory is 32 megabytes. We shall vary the cardinality of the join from 34 million to about 700 million, assuming that all tuples from both relations participate in the join. The performance of Jive-join,

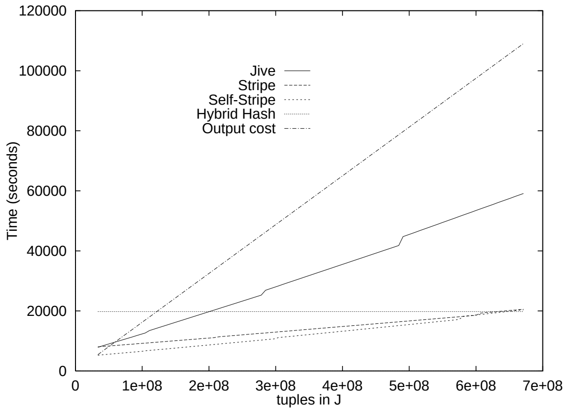


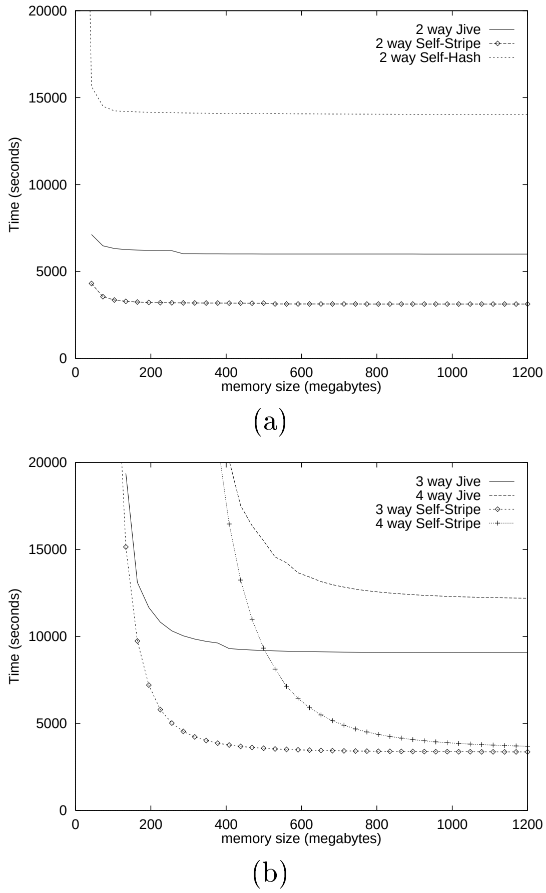
Figure 4: Performance graph for Example 5.2.

[5] The analytic cost formulas for Jive-join are reproduced in Appendix A.

Stripe-join, the self-join version of Stripe-join, and Hybrid-hash join are given in Figure 4. Also given is the output block-transfer cost. At the leftmost end of the scale Jive-join just beats Stripe-join. However, as the size of the join increases, Stripe-join dominates Jive-join by a substantial margin. For example, at 200 million join tuples Stripe-join takes time 11000 seconds, compared with 19800 for Jive-join (and Hybrid-hash join). That is a 17% saving in total cost when one includes the output cost. If this join happened to be a self-join, one can do even better. □

The other significant improvement of Stripe-join over Jive-join is apparent when one is performing a self-join.

Example 5.3: Suppose that we have a single relation of width 256 bytes containing 2^{25} tuples. We perform a join of that relation with itself, in which the join has size 2^{25} tuples and there is full participation of the relation in both arguments of the join. The performance of Jive-join, the self-join version of Stripe-join, and Self-hash join is given in Figure 5(a). When



(a)

(b)

Figure 5: Performance graphs for Example 5.3.

one takes into account the output cost of about 5450 seconds, the self-join version of Stripe-join improves on Jive-join by 24% in total time. By having to make only one pass through the participating relation Stripe-join can save over Jive-join. Because the intermediate hash files are relatively large for the Self-hash join, the overall cost is still high even though only one pass is made through the input relation.

We also perform a three-way and a four-way self-join of the relation, with similar join cardinality and tuple participation. The performance results appear in Figure 5(b). The differences become more dramatic as Jive-join needs to make three (respectively four) passes through the input relation rather than one. □

Improvements similar to those of Example 5.3 appear not just for self-joins, but also whenever a relation is repeated in a multi-way join.

Note that the optimization for self-joins does not apply to Jive-join because the two input relations to Jive-join are processed in separate phases of the Jive-join algorithm. Because Stripe-join treats all of its input relations symmetrically, we can "piggy-back" the work of later passes over one relation onto the first pass of that relation.

To be fair, the two-phase nature of Jive-join does allow a selection condition on a *nonindexed* attribute of the first relation to be combined within the join itself; such a combination is not straightforward with Stripe-join.

Finally, to understand the tradeoffs of using a join index, we examine the different algorithms' costs when the size of the join index is comparable to those of the input relations.

Example 5.4: Suppose the two input relations as well as the join result all have 2^{25} (about 34 million) tuples. Suppose that main memory is 32 megabytes and all tuples participate in the join. Given that the size of a join index record is 8 bytes, we shall simultaneously vary the tuple size of the input relations from 4 bytes to 40 bytes. The performance of Valduriez's

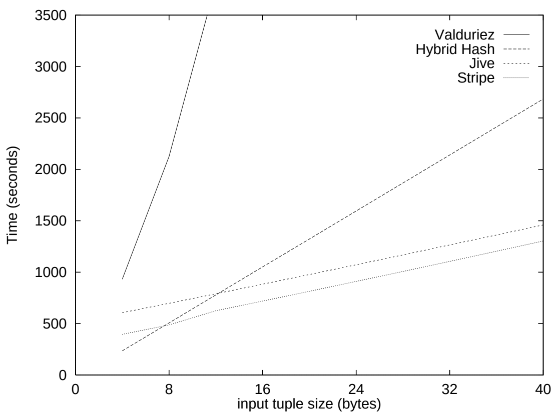


Figure 6: Performance graph for Example 5.4.

algorithm, Hybrid-hash join, Jive-join, and Stripe-join are given in Figure 6. Like in Example 5.1, Valduriez's algorithm performs the worst of all.

Stripe-join delivers slightly better performance than Jive-join throughout the test range as the former can dispense with the extra sorting step of the join index. Hybrid-hash join beats Jive-join when the input tuple size is less than 16 bytes, but beats Stripe-join only when the input tuple size is 4 bytes. As the size of the input tuples increases, the superiority of Jive-join and Stripe-join becomes evident. □

6 Extensions

Stripe-join can use multi-level recursion [7] when the inputs are larger than the memory bound (Equation 3). We need to perform additional levels of partitioning, and to make an intermediate partitioning pass over (some of) the input tuples. Thus, the cost of the algorithm will be higher. The details of this extension will be presented in the full version of this paper. In practice one would apply multi-level partitioning before hitting the memory bound of Equation 3 in order to reduce the number of seeks for small I/O units.

Stripe-join is particularly well-suited to joining inputs that are stored on tape. The input relations and the join index are accessed purely sequentially. Assuming that sufficient disk space was available for the temporary files and the output result, Stripe-join would be a good choice for performing the join with a pre-existing join index. Our cost model would have to be modified to model the characteristics of a tape drive.

There is a lot of potential parallelism in Stripe-join. Each of the input relations (and each of the segments within each relation) can, in principle, be processed independently. Compared with Jive-join, the parallelism in Stripe-join is more transparent since all relations are treated symmetrically. In Jive-join there are two distinct phases: one relation must be completely processed before the other relations can be processed.

7 Conclusions

We have proposed a new algorithm, Stripe-join, for performing a join using a join index.[6] Like its predecessor Jive-join from [12], Stripe-join has the following properties: Almost all of the I/O performed is sequential. A block of an input relation is read if and only if it contains a record that participates in the join. Skew does not adversely affect the performance. One can join multiple relations, retaining the single-pass property of the inputs, by using multidimensional data structures. For a wide variety of examples, the algorithm outperforms conventional algorithms including Valduriez's algorithm and Hybrid-hash join.

Stripe-join has important advantages over Jive-join:

- Jive-join requires a fixed ordering of the join index, while Stripe-join does not. Thus, Stripe-join can avoid a sorting step.

[6] We were informed by one of the referees that similar partitioning ideas have been used in the computation of Boolean functions by computing schemata [18].

- For joins in which a relation R appears more than once, Stripe-join can make a single pass through R to cover *all* of the instances of R in the join. In particular, self-joins can be performed with just one pass through the self-joined relation.

- Stripe-join delivers better performance under some circumstances, in particular when the cardinality of the join index is high.

To our knowledge Stripe-join is the first algorithm that, given a join index and a relation significantly larger than main memory, can perform a self-join with just a single pass over the input relation and without storing input tuples in intermediate files.

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A The Cost of Jive-join

For completeness, we present the Jive-join costs formulas from [12]. Let z' denote $(r-1)|J|/r$. The corresponding formulas for the performance of Jive-join given in [12] are[7]

$$\begin{aligned}
N_S &= \tfrac{3}{D}\left(|J| + |R_1| + \cdots + |R_r| + |J|/r\right) \\
&\quad +(r-2)y' + z'/v + n_1/x. \\
N_{I/O} &= Y(\tau_1||R_1||, |R_1|/\beta, ||R_1||) + \cdots + \\
&\quad Y(\tau_r||R_r||, |R_r|/\beta, ||R_r||) + z'/v' + n_1/x \\
&\quad +2(r-1)y' + |J|/\beta + . \\
N_X &= |J| + 2z' + \beta Y(\tau_1||R_1||, |R_1|/\beta, ||R_1||) \\
&\quad + \cdots + \beta Y(\tau_i||R_i||, |R_i|/\beta, ||R_i||).
\end{aligned}$$

n_1 denotes $||J|| * \ll R_1 \gg /b$ and the optimal values of x, y' and v' turn out to be

$$(x, v', y') = \left(\tfrac{m^r}{L(1+(r-1)\sqrt{z/n_1})}, \tfrac{m^r}{L((r-1)+\sqrt{n_1/z})}, \tfrac{L}{m^{r-1}}\right)$$

where $L = (|J|/r + \tau_2|R_2|)\cdots(|J|/r + \tau_r|R_r|)$.

Sorting the Join Index

When the join index is not clustered on one of its TID fields, we will have to sort the join index before Jive-join, Slam-join or Valduriez's algorithm can apply. We analyze the I/O cost for sorting a join index J. Since the I/O is mostly sequential, we shall ignore the seek and rotational latencies and just count the number of block transfers.

The sorting algorithm we use is a merge sort. We generate runs of length equal to the size of main memory, then merge those runs. (It is actually possible to generate runs that are twice as long as main memory [11], but the difference is immaterial in the present context.) The number of runs that are written to disk is $\lceil |J|/m \rceil$. On a single merge pass we can collapse n runs into $\lceil n/m \rceil$ runs. The total number of block transfers during sorting

[7] There is a small change in the value of N_S from [12]. Our estimate of N_S is slightly more accurate because it allows for only one dimension whose partition order corresponds to the join result order.

is then $2(1 + \lceil \log_m \lceil |J|/m \rceil \rceil)|J|$. (We can actually save up to $2m$ blocks by keeping the last run in memory, but the difference is not significant here.) This number needs to be added to the N_X value for any algorithm that requires a sorted join index.