Shopbots and Pricebots:

A Study of a Software Agent Economy

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February 16, 1999

Shopbot Economics

What are Shopbots?

- `www.acses.com`
- `www.shopper.com`
  - compares 1,000,000 prices on 100,000 products

Key Questions

- What is the future impact of shopbots on the electronic marketplace?
- What is the expected impact of agent technology on the nascent information economy?

Information Economies

George Stigler

- The Economics of Information [1961]
- examples of price dispersion (*e.g.*, used-car market)
- dispersion is attributed to costly search procedures

Shopbots Today

- shopbots specialize in collecting and distributing price information
- in global information economy, shopbots serve as localized marketplaces
- much like medieval markets that levied high taxes on sellers, shopbots charge vendors subscription, advertising, and commission fees

Pricebots Tomorrow

- automated agents that set prices in attempt to maximize profits for sellers, just as shopbots seek to minimize costs for buyers

Overview

Introduction

Seller' Decision Model

- Analysis
- Pricebot Simulations
- Discussion

Buyers' Decision Model

- Analysis
- Shopbot Simulations
- Discussion

Conclusion

Thesis

Sellers' Decision Model

single (homogeneous) commodity

S sellers, characterized by cost r_s

B buyers, characterized by valuation v_b

$B \gg S$ sellers' strategic interaction

Seller s's Decision choose price p_s

Profit Function $\pi_s(\vec{p}) = (p_s - r_s)D_s(\vec{p})$

where $\vec{p}$ is the vector of prices

and $D_s(\vec{p})$ is seller s's demand

Prescribed Buyer Behavior

Fixed Sample Size Search

Buyer of type i

- obtains i price quotes, at random
- selects a lowest-priced seller p_s
- purchases commodity iff $p_s \leq v_b$

Shopbots $\sim$ type S

Exogenous Buyer Distribution w_i

Game-Theoretic Analysis

Rationality

- sellers are profit maximizing
  - $p_s^* \in \arg\max \pi_s(p_s, p_{-s}^*)$

Nash Equilibrium

- no seller has any incentive to deviate
  - $\pi_s(p_s^*, p_{-s}^*) \geq \pi_s(p, p_{-s}^*)$, for all prices p

Analytical Results

1. no Nash equilibrium in pure strategies in general

2. derive symmetric mixed strategy Nash equilibrium

Pure Strategy Nash Equilibrium

Assumptions

- $v_b = v$, for all buyers b
- $r_s = r$, for all sellers s

Special Cases

- if $w_1 = 1$, then $p_s^* = v$, for all sellers s
- if $w_1 = 0$, then $p_s^* = r$, for all sellers s

Interesting Case $0 < w_1 < 1$

Pure Strategy Nash Equilibrium

Case $\quad 0 < w_1 < 1$ and $w_1 + w_S = 1$

Assume equilibrium vector of prices exists

1. at least one seller s charges $p_s^* < v$
   - because $\pi_s(p_s^*, v, \ldots, v) > \pi_s(v, v, \ldots, v)$
2. at most one seller s charges $p_s^* < v$
   - if $p_i^* = p_j^* < v$, then
     $$\pi_i(p_i^* - \epsilon, p_j^*, v, \ldots, v) > \pi_i(p_i^*, p_j^*, v, \ldots, v)$$
   - if $p_i^* < p_j^* < v$, then
     $$\pi_j(p_i^*, v, v, \ldots, v) > \pi_j(p_i^*, p_j^*, v, \ldots, v)$$
3. hence, at equilibrium, exactly one seller s charges $p_s^* < v$, while all other sellers charge v
4. now, v is a rational strategy, but p_s^* is not, since $p_s \in (p_s^*, v)$ yields strictly higher profits

No pure strategy Nash equilibrium exists

Symmetric Mixed Strategy Equilibrium

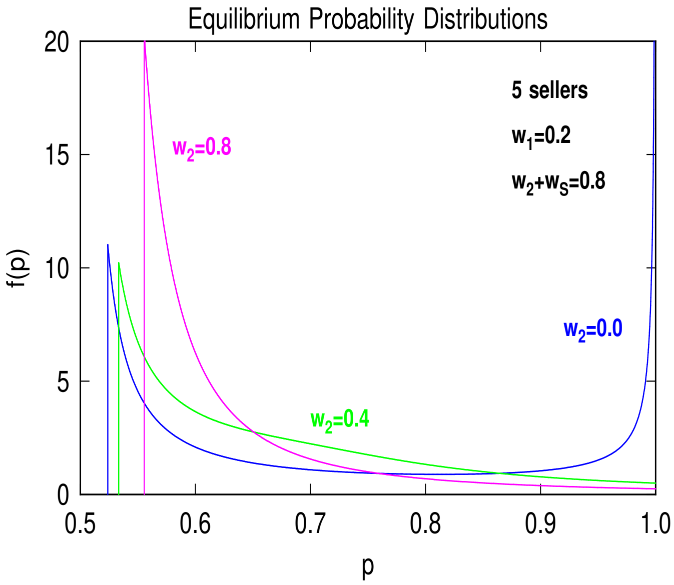
Probability Density Function $f(p)$

Cumulative Distribution Function $F(p)$

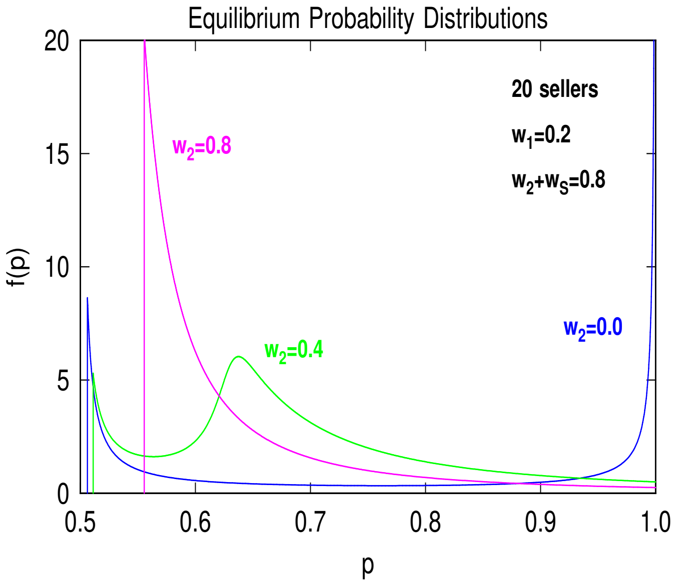
Properties

- $F(p)$ has no vertical discontinuities
  - since otherwise a seller could decrease its price by an arbitrarily small amount and experience a discontinuous increase in market share
- $F(p)$ has no horizontal discontinuities
  - since otherwise a seller charging a price at the low end of the plateau could increase its price to fill the gap while retaining its market share

PDFs



Equilibrium Probability Distributions
20
15
10
5
0
0.5
0.6
0.7
0.8
0.9
1.0
f(p)
p
5 sellers
w_1=0.2
w_2+w_S=0.8
w_2=0.8
w_2=0.4
w_2=0.0



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Pricebot Simulations

Information Economy of Computational Agents

- Shopbots seek prices that minimize costs for buyers
- Pricebots seek prices to maximize profits for sellers

Issues in Computational Learning Theory

- What sort of behaviors arise under simple adaptive pricing schemes?
  - given computational and informational constraints?
- Is equilibrium the outcome of adaptive learning, Nash or otherwise?

Learning Algorithms

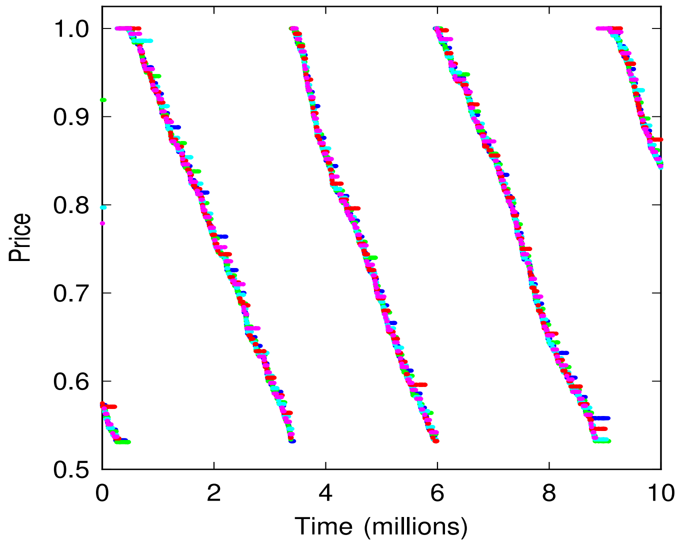
- MY myoptimal sellers
  - $p_s^{t+1} \in \arg\max_{p_s} \pi_(p_s, p_{-s}^t)$
  - high informational requirements
  - Cournot best-reply dynamics [1838]
- DF derivative followers
  - $p_s^{t+1} = p_s^t + \epsilon[\text{sign}(p_s^t - p_s^{t-1})\text{sign}(\pi_s^t - \pi_s^{t-1})]$
  - low informational requirements

Simulation Parameters

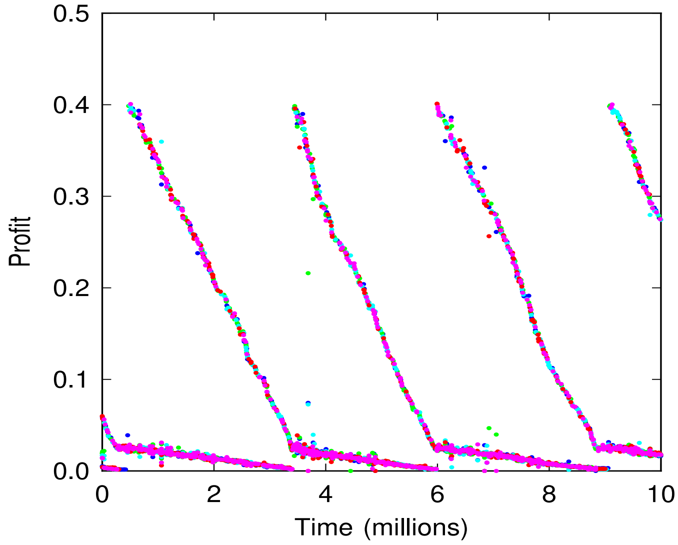
- 1000 buyers, 5 sellers
- $\rho_b = 20$, $\rho_s = 1$
- $w_1 = 0.25$, $w_S = 0.75$
- $v = 1$, $r = 0.5$
- 10 million iterations

Myoptimal Sellers

Price Wars



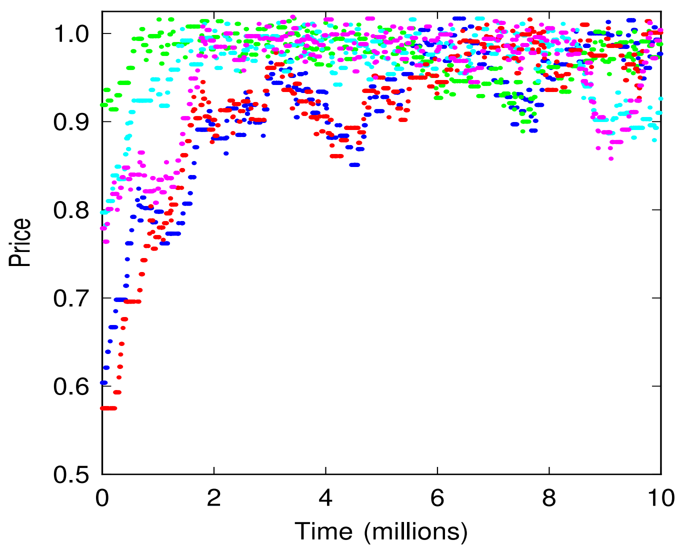
5 MY sellers, w_B=0.75



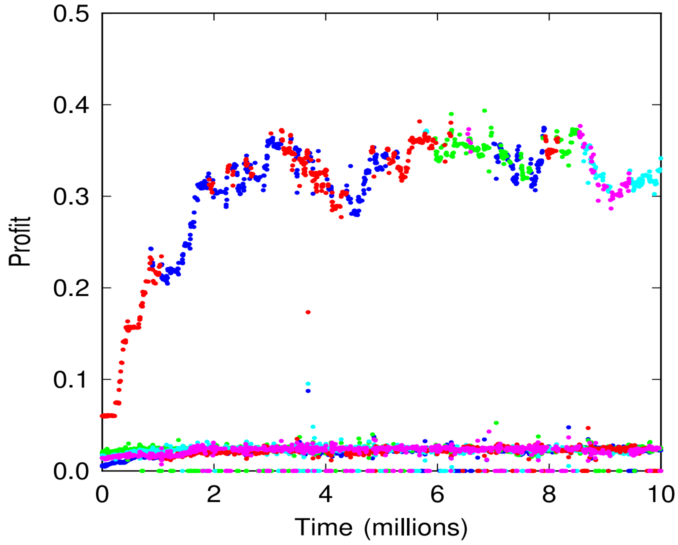
5 MY sellers, w_B=0.75

Derivative Followers

Tacit Collusion

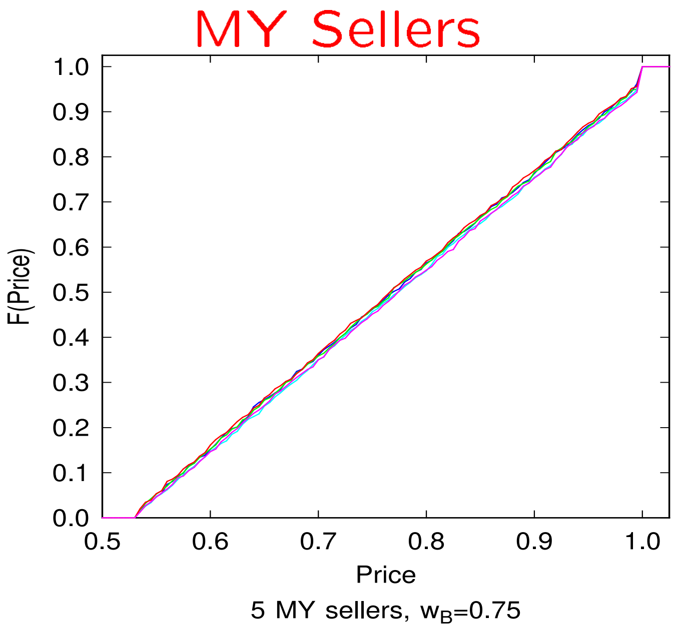


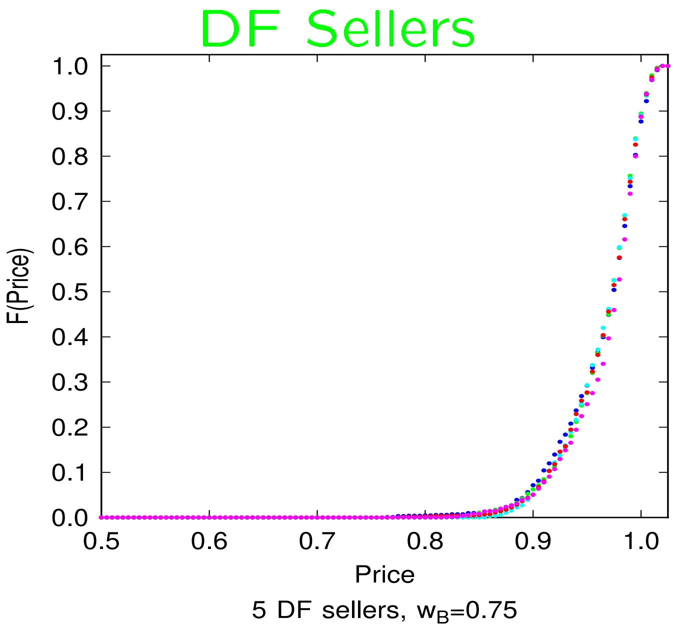
5 DF sellers, w_B=0.75



5 DF sellers, w_B=0.75

CDFs


MY Sellers
F(Price)
Price
5 MY sellers, w_B=0.75


DF Sellers
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Price
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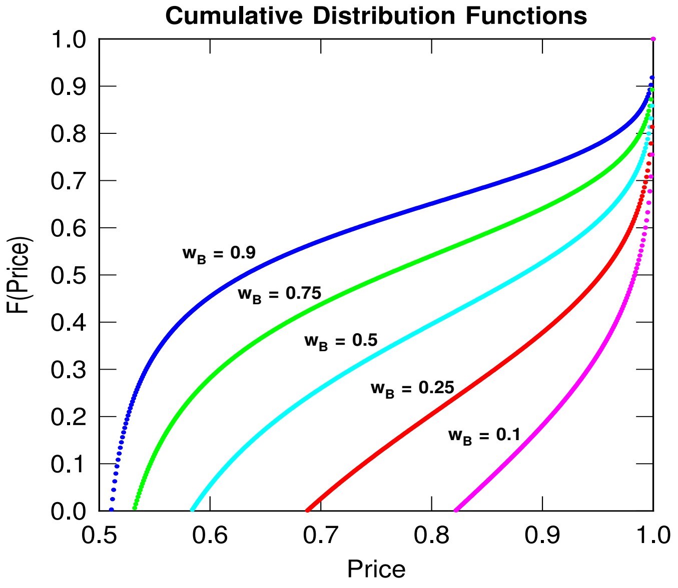
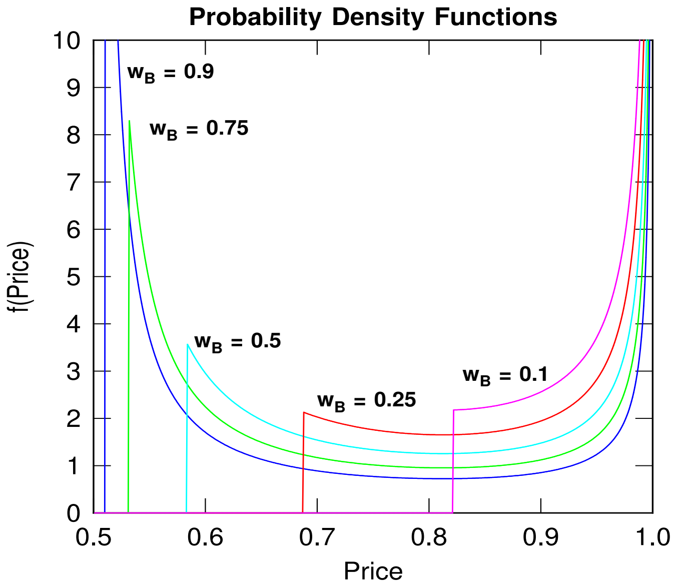
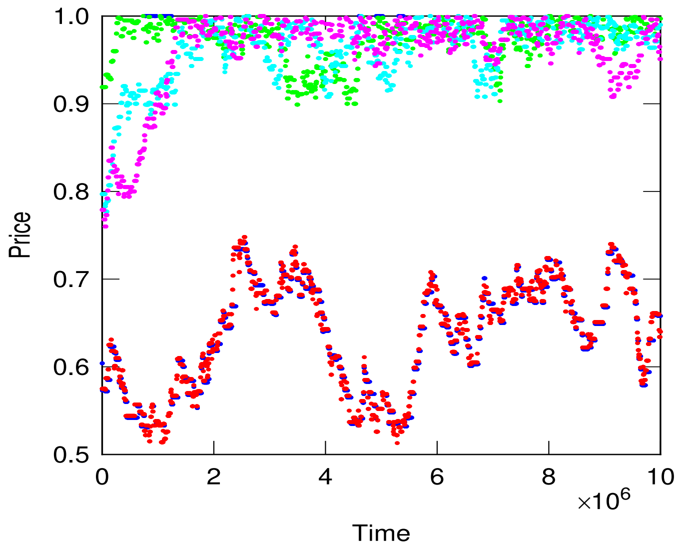
Nash Equilibria

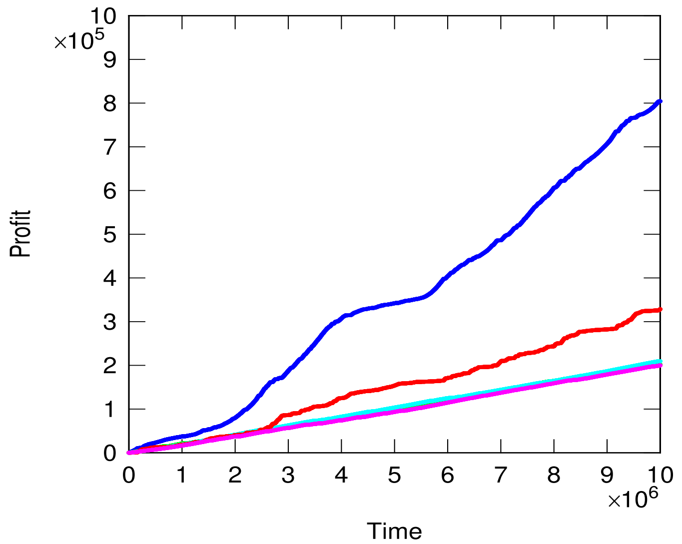




1 MY vs. 4 DF

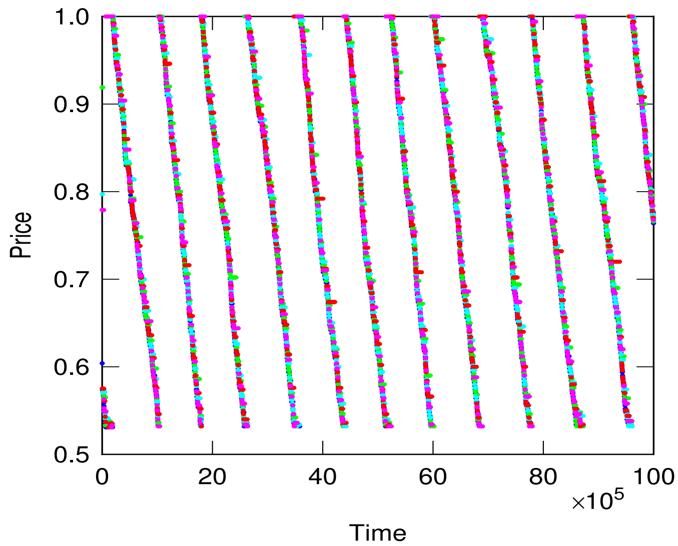


1 MY + 4DF sellers, w_A=0.25, w_B=0.75

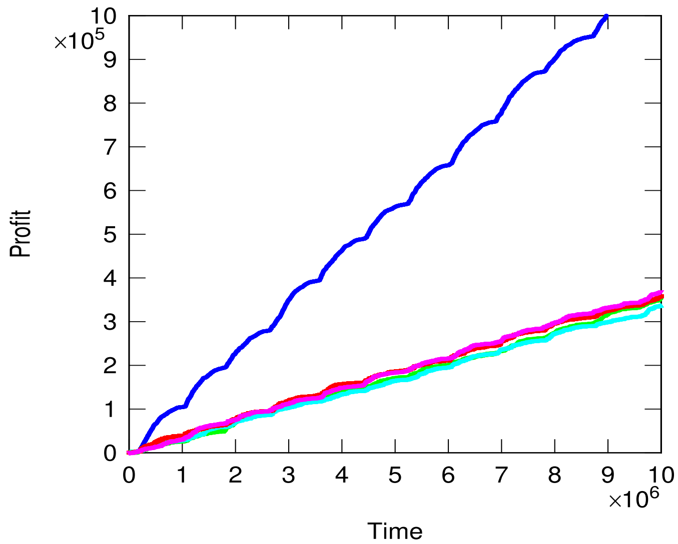


1 MY + 4 DF, w_A=0.25, w_B=0.75

1 Fast MY vs. 4 MY



1 Fast MY + 4 MY, w_A=0.25, w_B=0.75



1 Fast MY + 4 MY, w_A=0.25, w_B=0.75

Discussion

Evolution of Bots

- it pays for sellers to build smart and fast pricebots
  - $\Rightarrow$ pricebots obtain pricing information from shopbots
  - $\Rightarrow$ current shopbot business model, based on advertising revenues, becomes ineffective
- Internet meltdown
  - $\Rightarrow$ shopbots charge users for pricing information

Value of Information

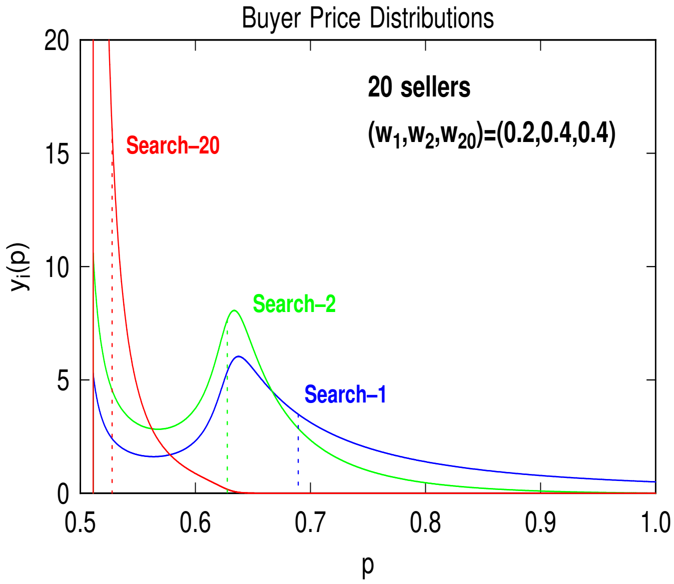
Buyer Price Distributions

CDF for the minimum of i independent samples

$$Y_i(p) = 1 - [1 - F(p)]^i$$

PDF for the minimum of i independent samples

$$y_i(p) = i f(p)[1 - F(p)]^{i-1}$$



Buyers' Decision Model

cost c_i of obtaining i (random) price quotes

buyers estimate price $\widehat{p}_i$, given i price quotes

Buyer Decision Problem choose type i

Expected Cost Function $\widehat{p}_i + c_i$

Endogenous Buyer Distribution w_i

Exogenous Cost Function $c_i = c_1 + \delta(i - 1)$

Game-Theoretic Analysis

Rationality

- buyers are cost minimizing
  - $i^* \in \arg\min_{1 \leq i \leq S} \widehat{p}_i + c_i$
  - $\widehat{p}_i = \bar{p}_i$ average price

Nash Equilibrium

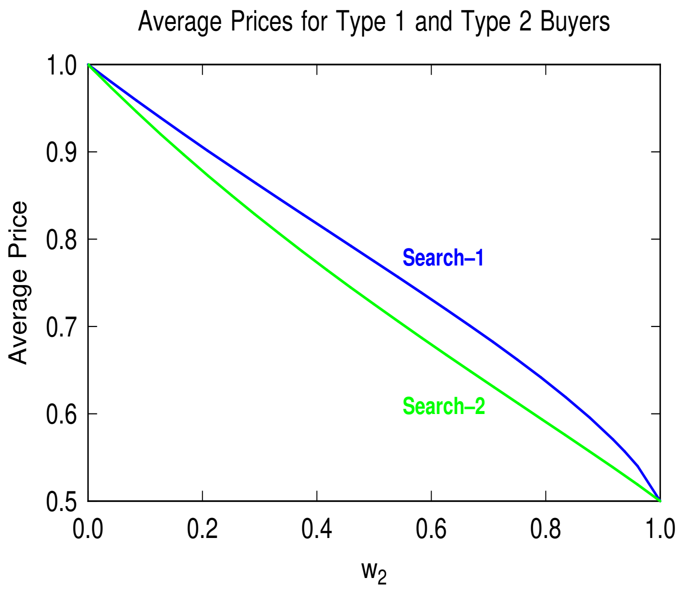
- at equilibrium, $0 < w_1 \leq 1$, since if $w_1 = 0$, then $p_s^* = r$, for all sellers s, in which case $w_1 = 1$
- the cost function $\bar{p}_i + c_i$ is convex, because as i increases, $\bar{p}_i$ decreases, at a non-increasing rate
- minimized at either a single integer value i^* or two consecutive integer values i and $i^* + 1$

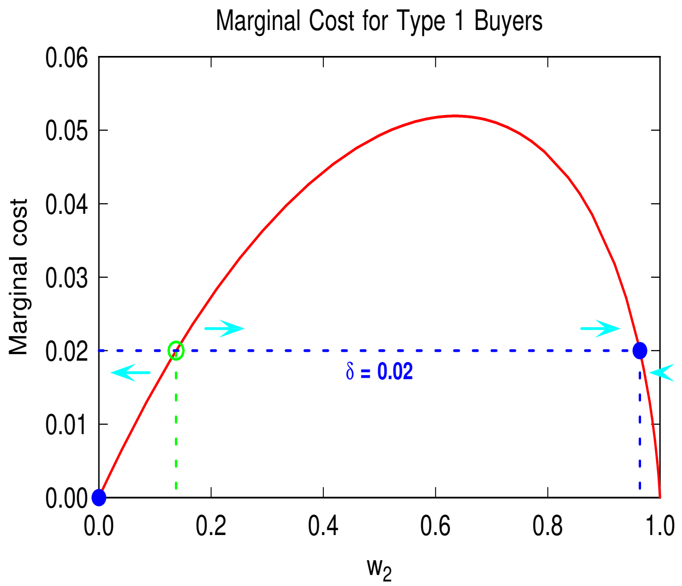
Multiple Equilibria

1. $w_1 = 1$ and $p_s^* = v$, for all sellers s
2. $w_1 + w_2 = 1$ and sellers play mixed strategies $f(p)$

Burdett and Judd, 1983

Buyers of Type 1 and 2





Shopbot Simulations

Buyer Population

- $w_i^{t+1} = w_i^t(1-\epsilon) + \epsilon\delta_{ii^*}$

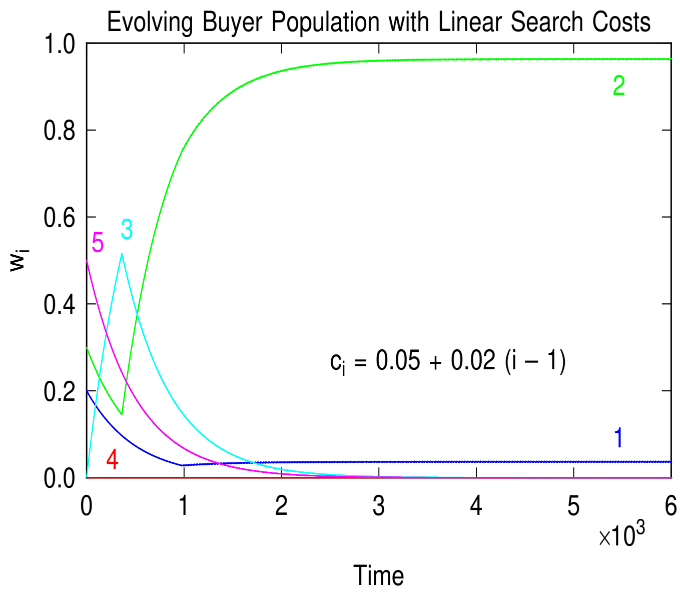
Seller Population

- plays game-theoretic equilibrium

Shopbot Experiments

- linear cost function
- non-linear cost function
- set floor value for w_1

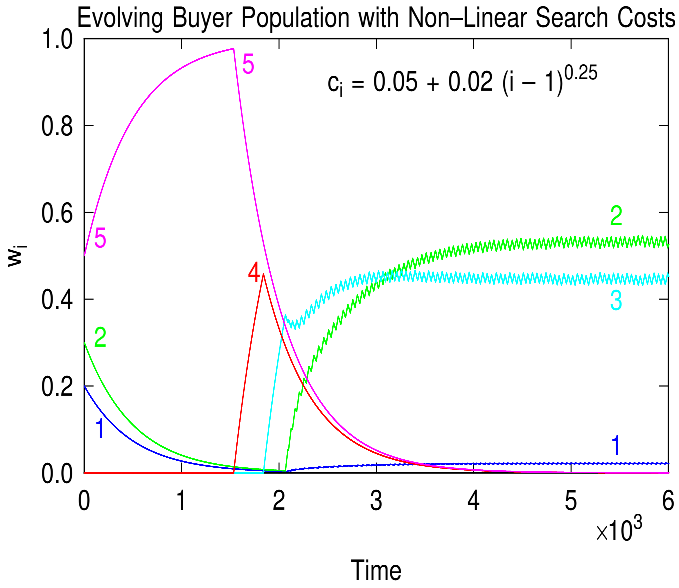
Shopbot Simulations



Observations

- linear cost function $c_i = 0.05 + 0.02(i - 1)$
- buyer strategy vector evolves to $w_1 + w_2 = 1$
- final state $(w_1, w_2) = (0.036, 0.964)$

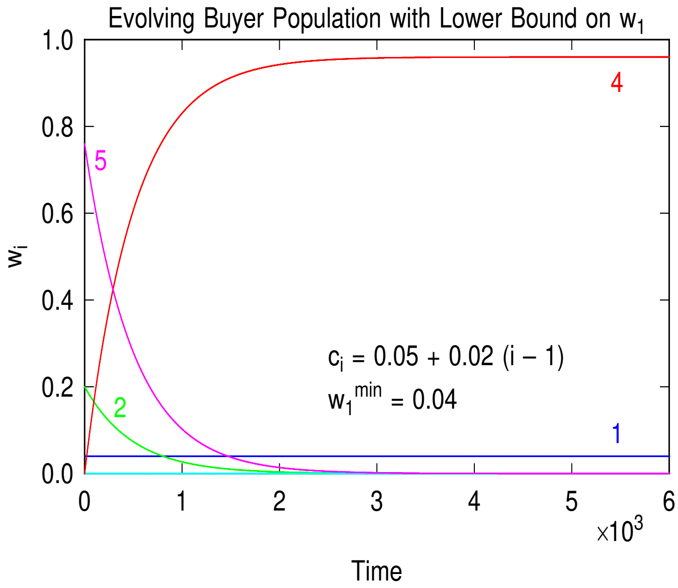
Shopbot Simulations



Observations

- non-linear cost function $c_i = 0.05 + 0.02(i-1)^{0.25}$
- buyer strategy vector evolves to $w_1 + w_2 + w_3 = 1$

Shopbot Simulations



Observations

- set floor value for $w_1 = 0.04$
- buyer population evolves to $w_1 + w_i = 1$
- i increases with $\lfloor w_1 \rfloor$ and decreases with δ

$\lfloor w_1 \rfloor$	$\delta = 0.001$	$\delta = 0.005$	$\delta = 0.020$
0.01	5	2	2
0.04	5	2–5 *	2
0.20	5	5	2–3 †

Summary and Conclusions

- game-theoretic model and analysis of shopbot economics
- pricebot simulations lead to seller price wars and tacit collusion
- shopbot simulations evolve towards enhanced buyer search
- value of information in electronic markets

Future Work

- Study the dynamic interplay of adaptive buyers and sellers

- Study dynamic pricing of information in a full-fledged economy of software agents, w/ electronic buyers, sellers, and shopbots, where the latter act as intermediaries, strategically pricing the information services they provide

- further study of computational learning of game-theoretic equilibria in shopbot economics

Thesis

Methodology

- computational game theory
- learning algorithms

Applications

- Networking Games
  - Nash equilibrium is not generally the outcome of learning in network settings
  - with Scott Shenker and Eric Friedman
- Santa Fe Bar Problem
  - a negative result on learning, rational learning does not converge to Nash equilibrium
  - with Rohit Parikh and Bud Mishra

References

http://www.cs.nyu.edu/amygreen

http://www.research.ibm.com/infoecon