Welcome to Discrete Math!

Start of Lecture 1

Today: Objectives, Propositions and Predicates, Syllabus

An exercise

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(or someone messed up)

Course Objectives

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- Use mathematical language to precisely communicate.
- Think mathematically and prove statements in a variety of domains.
- Develop a toolkit of definitions and techniques from graph theory, combinatorics, probability, and number theory.

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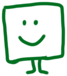

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- x is divisible by 7.

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Example: Let $P(n)$ be the predicate 'n is divisible by 7'.
Then $P(14)$ is true, while $P(15)$ is false.

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Notation you might see:

- $\forall x \in \mathbb{N}, x^2 \geq x$.
- $\forall x \in \mathbb{R}, x^2 \geq x$.
- $\exists x \in \mathbb{R} : x^2 = 2$.
- $\exists x \in \mathbb{Z} : x^2 = 2$.

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  Styopa is **not** 6 feet tall.
  $\neg$("The sky is blue")

Logical Operators: And, Or, Not

Truth tables for and, or, and not:

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p	q	$p \wedge q$
T	T	
T	F	
F	T	
F	F	

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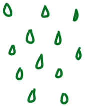
Implies/If then: (denoted $\implies$)
"It's raining $\implies$ the ground is wet."



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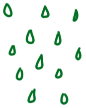

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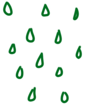

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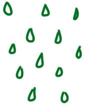

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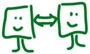

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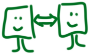

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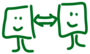

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Syllabus Highlights

Grade Breakdown:

- Homework: 25%
- Midterm 1: 20%
- Midterm 2: 20%
- Final Exam: 35%
- Recitations: 4% Extra Credit

Syllabus Highlights

Homeworks:

- Weekly, due on Fridays at 11:59 PM.
- Collaboration is encouraged, but you must write your own solutions, list collaborators.

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More details on course website...