Recitation 7: Eulerian paths, isomorphism

1 Eulerian paths



Figure 1: Dominos laid out in a ring, where adjacent numbers match (thank you google images)

Example 1.1. A domino set has $\binom{7}{2}$ dominos, each of which has two sides with numbers from 0 to 6 (no repeats: no domino has the same number twice). Can you lay all of them out in a closed ring, so that touching ends match, as in Fig. 1?

Styopa's section did this in class. (We didn't have time to get to this in Gabriel's section). The domino problem is equivalent to finding an Eulerian cycle in K_7. We will go over this in recitation.

Problem 1. What if my domino set has sides that go from 0 to 5 instead?

Problem 2. What if my domino set has sides that go from 0 to 5, but is missing the dominos $(0,1),(1,2),(2,3),(3,4),(4,5),(5,0)$?

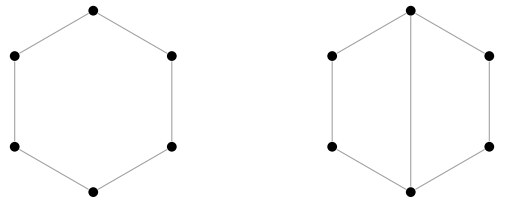
Problem 3. What if my domino set has sides that go from 0 to 5, but is missing the dominos $(0,1),(2,3),(4,5)$?

2 Isomorphism

Problem 4. List all the different graphs on 4 vertices, where two graphs are "the same" if they are isomorphic.

Problem 5. For each pair below, decide whether the two graphs are isomorphic. If they are, give the bijection. If they are not, say which invariant tells them apart.

(a)



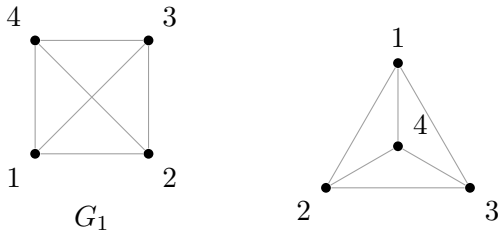
G_1 G_2

(b)



G_1 G_2

(c)



G_1 G_2

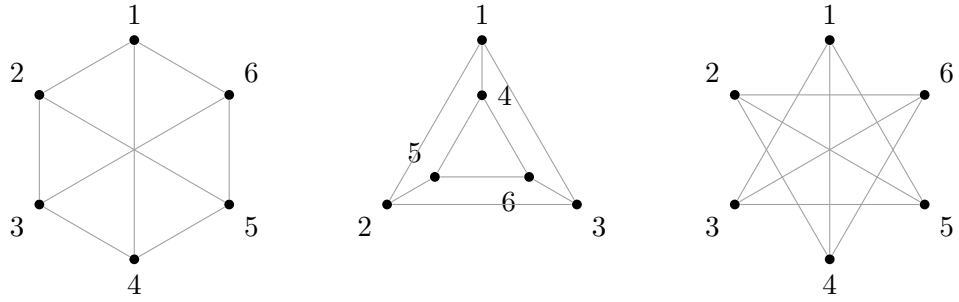
Problem 6. For each pair below, decide whether the two graphs are isomorphic. If they are, give the bijection. If they are not, give an invariant that tells them apart.

(a) Two trees:



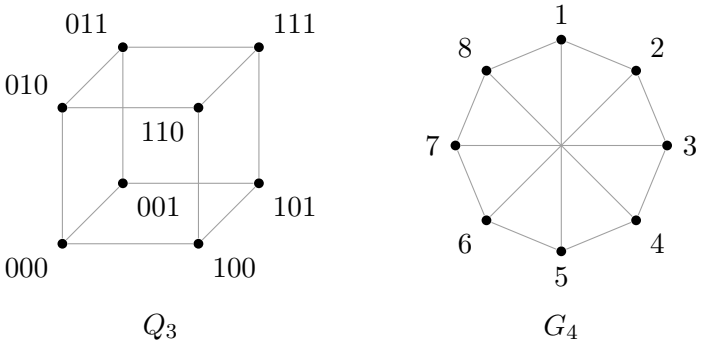
G_1 G_2

(b) Three graphs. Which pairs are isomorphic?

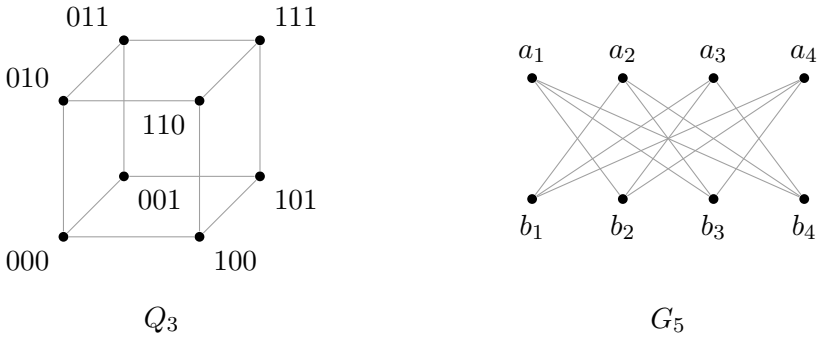


G_3 is the complement of the 6-cycle $1, 2, 3, 4, 5, 6$.

(c) The cube Q_3, and an octagon with its four long diagonals:



(d) (hard) The cube Q_3 again, and $K_{4,4}$ with 4 edges deleted:



G_5 has an edge from a_i to b_j whenever $i \neq j$. So the only missing edges are a_1b_1, a_2b_2, a_3b_3 and a_4b_4.