Recitation 6: Counting in Two Ways, Graphs, and Pigeonhole

You may leave answers unsimplified, e.g. in terms of $\binom{n}{k}$ and factorials.

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1 Counting in Two Ways

Problem 1. Using counting in two ways, show that

$$\binom{n}{3} = \sum_{i=1}^{n} (i-1)(n-i).$$

Problem 2. Using counting in two ways, show that

$$\sum_{i=0}^{n} \binom{i}{k} = \binom{n+1}{k+1}.$$

2 Graph Theory

Problem 3. For each of the following sequences, either draw a graph whose vertices have those degrees, or prove that no such graph exists.

(a) $1, 2, 2, 3, 3, 4$

(b) $4, 4, 4, 4, 4$

(c) $1, 1, 1, 1, 1$

(d) $5, 3, 2, 2, 1, 1$

(e) $3, 3, 3, 3, 3$

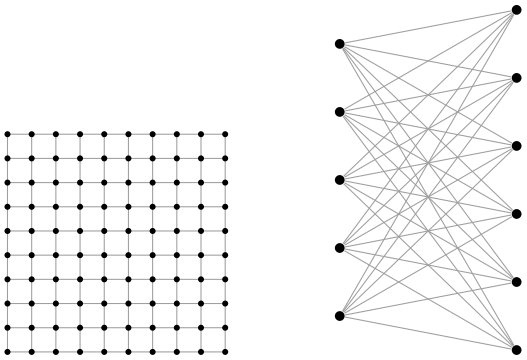
(f) $5, 5, 1, 1, 1, 1$

Problem 4.

(a) How many 3-cliques are there in K_n?

(b) What is the largest independent set in the 10×10 grid graph, drawn below? Prove that it is the largest.

(c) What is the largest independent set in $K_{5,6}$, drawn below? Prove that it is the largest.



3 Pigeonhole principle

Problem 5. Chris picks 51 numbers from [100]. Prove that no matter which numbers he picks, two of them sum to 101.