Recitation 5: Counting

You may leave answers unsimplified, e.g. in terms of $\binom{n}{k}$ and factorials.

Problem 1. In poker, a *three of a kind* is a 5-card hand with three cards of one rank, and two more cards of two different ranks (different from each other, and from the rank of the triple). For example, $\{7\heartsuit, 7\spadesuit, 7\clubsuit, 2\diamondsuit, J\clubsuit\}$ is a three of a kind.

(a) How many three of a kind hands are there?

(b) In lecture we counted *two pair* hands, and had to divide by 2 at the end. Did you do something similar here? If so, why? If not, why not?

Problem 2.

(a) How many surjective functions are there from $[10]$ to $[3]$?

(b) How many surjective functions are there from $[n]$ to $[3]$, for $n \geq 1$?

Problem 3. I have 15 identical candies to give to 4 trick-or-treaters.

(a) How many ways are there to do this?

(b) How many ways are there if no one may get more than 6 candies?

Problem 4. Let $n \geq 1$. Prove that $[n]$ has the same number of odd-sized and even-sized subsets.

> *Hint*: don't count directly; instead, construct a bijection from odd-size to even-size subsets.