Recitation 2: Induction

A Bad Proof

Claim. For all n, any group of n horses are all the same color.
"Proof." We induct on n.
Base case. $n = 1$: any group of 1 horse is obviously all the same color.
Inductive step. Fix k, and assume that any group of k horses are all the same color (the induction hypothesis). Consider a group of $k+1$ horses. Remove any one horse; the remaining k horses are all the same color, by IH. Put that horse back, and remove a different one; the remaining k horses are *also* all the same color, by IH. So all $k+1$ horses are the same color.
What's wrong with this proof?

Induction Practice Problems

Proposition. $\displaystyle\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$.

Proposition. A chocolate bar with n squares requires exactly $n-1$ breaks to separate into individual squares, no matter what order/how you break them.

Proposition. Find a closed form for $\displaystyle\prod_{i=2}^{n}\left(1-\frac{1}{i^2}\right)$, and prove it.

Proposition. The interior angles of an n-sided polygon sum to $180(n-2)$ degrees.

Proposition. Let F_n denote the nth Fibonacci number. Then $F_n \leq 1.7^n$.