Recitation 1: Proofs

Negating Statements

Practice negating statements. Here are some examples to negate:

- For all students x, x is tall or x is short.
- There is a largest integer. How would you write this using quantifiers?
- All integers are either even or odd.

$$\forall n \in \mathbb{Z},\ \big(\exists k \in \mathbb{Z} : n = 2k\big) \text{ OR } \big(\exists l \in \mathbb{Z} : n = 2l + 1\big).$$

Practice Proofs

(Easy)

Proposition. Let $r \in \mathbb{R}$. If r is irrational, then $r^{1/5}$ is irrational. (by contrapositive)

Proposition. If $n \in \mathbb{N}$, then n^2 leaves a remainder of 0 or 1 when divided by 4. (by cases)

Proposition. If $p \mid q$ and $q \mid r$ then $p \mid r$. (direct)

(Medium)

Proposition. Assume $\sqrt{6}$ is irrational. Show that $\sqrt{2} + \sqrt{3}$ is irrational. (by contradiction)

Proposition. Let $n \in \mathbb{N}$. If $n > 2$ and n is prime, then n is odd. (by contradiction, assuming an even prime > 2, or by contrapositive: if n is even, then $n \leq 2$ or n is not prime.)

Proposition. For $a, b \in \mathbb{Z}$: $a \mid b$ and $b \mid a$ if and only if $a = b$ or $a = -b$.
Remember that an "if and only if" needs to be proven in both directions!

(Hard)

Using the following identities about 0, 1, and -1:

1. For all $x \in \mathbb{R}$, $1x = x$. (This is the definition of 1.)
2. For all $y \in \mathbb{R}$, $0 + y = y$. (This is the definition of 0.)
3. $-1 + 1 = 0$. (This is the definition of -1.)

(a) Prove that for all z, $z \cdot 0 = 0$.

(b) Prove that $(-1) \cdot (-1) = 1$.