Recitation 0: Logic, truth tables, even/odd

Sep 11, 2026

Practice Problems

1. Let P be the set of prime numbers. Write, using logical operators and quantifiers, the statement that "every prime number greater than 2 is odd." Can you write this two different ways?

2. Prove the following equivalences using a truth table:

$$\begin{aligned} \neg\neg p &\equiv p \\ p \implies q &\equiv \neg p \vee q \\ p \implies q &\equiv \neg q \implies \neg p \qquad \text{(contrapositive)} \end{aligned}$$

   Prove the following tautologies using a truth table. A tautology is a logical expression that always evaluates to true, no matter what the arguments (p, q) are.

$$\begin{aligned} p \vee \neg p &\equiv \text{true} \\ ((p \implies q) \wedge p) \implies q &\equiv \text{true} \qquad \text{(modus ponens)} \\ p \wedge q \implies p &\equiv \text{true} \\ \text{false} \implies (p \wedge \neg p) &\equiv \text{true} \end{aligned}$$

3. Decide for each of the following which are true, where P and Q are arbitrary predicates.

   (a) $\forall x\, (P(x) \wedge Q(x)) \overset{?}{\iff} \forall x\, P(x) \wedge \forall x\, Q(x)$

   (b) $\forall x\, (P(x) \vee Q(x)) \overset{?}{\iff} \forall x\, P(x) \vee \forall x\, Q(x)$

   (c) $\exists x\, (P(x) \vee Q(x)) \overset{?}{\iff} \exists x\, P(x) \vee \exists x\, Q(x)$

   (d) $\exists x\, (P(x) \wedge Q(x)) \overset{?}{\iff} \exists x\, P(x) \wedge \exists x\, Q(x)$

4. When we factor a quadratic, we often do something like

$$(x-a)(x-b) = 0 \implies x = a, b,$$

   but then there's sometimes an extraneous solution. Why? What is the fact that we're using here?

   Similarly, when "we square root both sides," we often do something like

$$x^2 = b \implies x = \pm\sqrt{b}.$$

   What's the exact fact here?