COMS 3203: Discrete Math, Fall 2026

Problem Set 7: Probability and Graph Theory 2

Due on Tuesday, November 24, 2026

Note: you may leave all expressions in unsimplified form, i.e. using fractions, $\binom{n}{k}$, sums, products, exponents, etc. (If you wish, you may also compute the decimal representation of probabilities).

For all questions, prove/explain your answer (i.e., show your work).

Problem 0 Please put your answers to problem 0 on a different page/image when uploading, so that we don't see your name when grading.

(a) Who did you work with on this problem set?

(b) How long did you spend on this problem set?

Problem 1 Michael has a bin with 5 blue candies, 10 red candies, and 20 green candies. He reaches in and take two candies at random. What is the probability that the two candies are the same color?

Problem 2 Chris rolls a 10-sided dice (which has faces 1 through 10) two times. What is the probability that the first roll is 10 or the total of the two rolls is odd?

Problem 3 William chooses an integer x uniformly at random from [30]. Which pairs of the following events are independent, and which are dependent?

$A : x$ is divisible by 2. $B : x$ is divisible by 3. $C : x$ is divisible by 7.

Problem 4 Recall that $K_{m,n}$ (for $m, n \geq 1$) is the complete bipartite graph.

- A path or cycle is *Eulerian* if it uses every edge of the graph exactly once.
- A path or cycle is *Hamiltonian* if it visits every vertex of the graph exactly once (except for the start/endpoint, in the case of the cycle).

(a) Under what conditions (i.e., for what values of m and n) does $K_{m,n}$ have an Eulerian path?

(b) Under what conditions does $K_{m,n}$ have a Eulerian cycle?

(c) Under what conditions does $K_{m,n}$ have a Hamiltonian path?

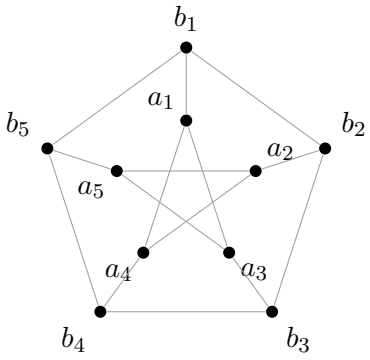
(d) Under what conditions does $K_{m,n}$ have a Hamiltonian cycle?

A short explanation suffices for each.

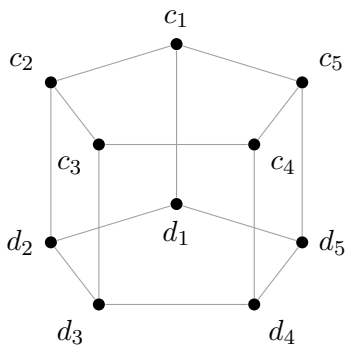
Problem 5 Which of these three graphs G_1, G_2, G_3 are isomorphic?

For any pair that is isomorphic, show a bijection by drawing one graph labeled with the vertices of the other (for example, if you think that G_1 and G_2 are isomorphic, draw G_1 and label its vertices $c_1 \cdots c_5, d_1 \cdots d_5$).

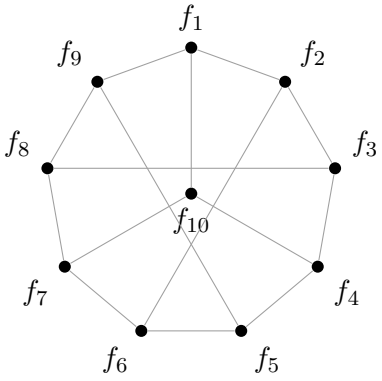
For any pair that are not isomorphic, give a property that one has that the other does not.



G_1



G_2



G_3