COMS 3203: Discrete Math, Fall 2026

Problem Set 5: Counting

Due on Tuesday October 27, 2026

Note: you may leave all expressions in unsimplified form, i.e. using factorials, $\binom{n}{k}$, sums, products, exponents, etc. However, you *should* explain where your answers are from, e.g. via a sequential decision procedure, partition, overcounting, etc.

Problem 0 Please put your answers to problem 0 on a different page/image when uploading, so that we don't see your name when grading.

(a) Who did you work with on this problem set?

(b) How long did you spend on this problem set?

Problem 1 In poker, a *full house* is a set of 5 cards that consists of a 3 of a kind and a 2 of a kind. For example, the hand $\{A\heartsuit, A\spadesuit, A\diamondsuit, 5\diamondsuit, 5\clubsuit\}$ is a full house. How many distinct full houses are there?

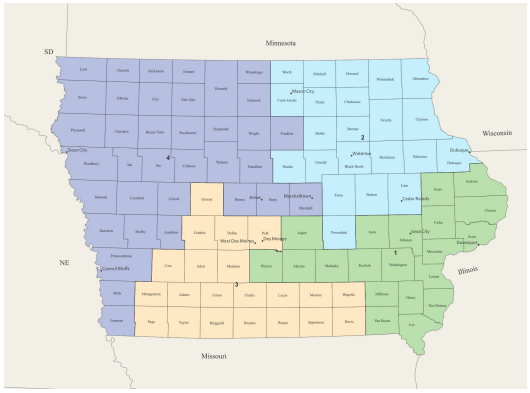
Problem 2 How many functions $f : [5] \to [5]$ have exactly 2 elements in their image?

Problem 3 Isa, Keona, Sindhu, and Alex want to divide up 20 apples and 10 oranges such that every TA gets at least one fruit, all fruits are assigned, and no TA gets multiple types of fruit. How many ways are there to do this?

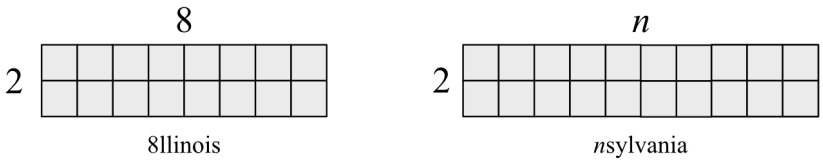
Problem 4 Olivia's street has 4-digit house numbers going from #0000 to #9999. A house number is *neat* if the digits add up to 18 (for example, house #1917 is neat). A house number is *tidy* if the sum of the first two digits equals the sum of the last two digits (for example, house #3865 is tidy). Show that there are the same number of neat and tidy houses.

Hint: Instead of counting the number of neat and tidy house numbers, try to construct a bijection between them. Can you turn a neat house number into a tidy house number by changing some of its digits?

Problem 5 Each state in the US consists of n counties. Once every ten years or so, each state does a process called *redistricting*, where the counties are grouped into k contiguous pieces, called *districts*, where each district elects one person to Congress. For example, Iowa has 99 counties, which are divided into 4 districts:

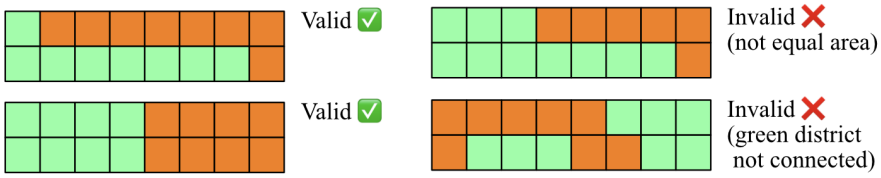


In this problem, we'll consider the question of counting how many districtings there are of some simple states: 8llinois, which has 16 counties in a 2×8 grid, and $n$sylvania, which has $2n$ counties in a $2 \times n$ grid.



(a) How many ways are there to divide 8llinois into 2 districts of equal size? Note that the actual "color" or "label" of the districts doesn't matter (i.e., if you flip the colors, it's not a different districting).

For example, here are some valid and invalid districtings of 8llinois:



(b) How many ways are there to divide $n$sylvania into 2 nonempty districts if they don't have to be the same size?

(c) How many ways are there to divide $n$sylvania into n districts of size 2?

Hint: Think about defining it in terms of a recursive sequence. How many districtings are there of 1sylvania? How many districtings are there of 2sylvania?

In general, for arbitrary states, this problem is very difficult. For example, no one knows how many equal-size districtings there are of a 20×20 grid; if you manage to prove an expression for this, someone will probably give you a PhD on the spot.