COMS 3203: Discrete Math, Fall 2026

Problem Set 4 (short): Countability

Due on Tuesday October 20, 2026

Problem 0

(a) Who did you work with on this problem set?

(b) How long did you spend on this problem set?

Problem 1 Congratulations on getting through midterm 1! How are you feeling about the class so far? Take a few minutes to think about how it's all going. Tell us your thoughts in this form. (This question is *not* optional!)

Problem 2 Suppose that A and B are disjoint finite sets (that is, $A \cap B = \varnothing$). Prove that if $|A| = n$ and $|B| = m$, then $|A \cup B| = m + n$, using the definition of the size of a set (i.e., using bijections).

Problem 3

(a) State a bijection f from $\mathbb{E}$ to $\mathbb{N}$, where $\mathbb{E} = \{2, 4, 6, 8, \cdots\}$ is the set of even natural numbers.

(b) In class, we constructed a bijection b between $\mathbb{N}$ and $\mathbb{Z}$ via

$$b(n) = \begin{cases} n/2 & \text{if } n \text{ even} \\ -(n-1)/2 & \text{if } n \text{ odd.} \end{cases}$$

Combine that with your bijection f to make a bijection $g : \mathbb{E} \to \mathbb{Z}$.

(c) Prove that g is bijective by finding a function $g^{-1} : \mathbb{Z} \to \mathbb{E}$ that is a two-sided inverse for g. Prove that g^{-1} is a two-sided inverse by showing that $g(g^{-1}(x)) = x$ and $g^{-1}(g(x)) = x$ for all x.