COMS 3203: Discrete Math, Fall 2026

Problem Set 2: Proofs and Induction

Due on Tuesday September 29, 2026

Problem 0

(a) Who did you work with on this problem set?

(b) How long did you spend on this problem set?

Problem 1

(a) Prove that if $a, b \in \mathbb{Z}$ are solutions to $a^3 + ab^2 + b^3 = 0$, then a and b are both even.

(b) Using the above, prove that there is no rational number x such that $x^3 + x + 1 = 0$.

Problem 2 Proposition. If $n \in \mathbb{N}$ is odd, then $n^2 - 1$ is divisible by 8.

(a) Prove the proposition without using induction.

(b) Prove the proposition using induction.

Problem 3 Arav and Selin run a candy store that sells lollipops in packs of 3 and packs of 5. For which natural numbers n can you buy exactly n lollipops? (For example, you can get 16 lollipops by buying two packs of 5 and two packs of 3). Prove your claim.

Problem 4 Prove that every natural number can be expressed as the sum of distinct powers of 2. For example, $42 = 32 + 8 + 2 = 2^5 + 2^3 + 2^1$.

Problem 5 Brandon and Khine play the following game: starting with a bag of n marbles, they take turns removing either 1, 2, 3, or 4 marbles from the bag. Whoever removes the last marble wins. Brandon goes first. For which values of n does Khine have a strategy to win regardless of what Brandon does? Prove your claim.

(Hint: start with small values of n and work upwards.)

Problem 6 (optional, 1 bonus point) Let f_i be a sequence defined as

$$f_0 = 0, \quad f_1 = 1, \quad \text{and} \quad f_n = f_{n-1} + f_{n-2} \text{ for } n \geq 2.$$

Prove that for all $n \in \mathbb{N}$, $f_n^2 + f_{n+1}^2 = f_{2n+1}$ and $f_{n+2}^2 - f_n^2 = f_{2n+2}$.

(Hint: it may help to sometimes write $f_{n+1} = f_{n+2} - f_n$.)