COMS 3203: Discrete Math, Fall 2026

Problem Set 1: Logic and Proofs

Due on Tuesday September 22, 2026

Problem 0
(a) Who did you work with on this homework?
(b) How long did you spend on this homework?

Problem 1 Prove the following statement:

$$\forall x \in \mathbb{Z}, \quad x \text{ is even} \iff 5x + 3 \text{ is odd.}$$

Problem 2
(a) Prove that the statement "$\forall x \in \mathbb{Z}, \exists y \in \mathbb{Z}, x + y < 0$" is true.
(b) Prove that the statement ""$\exists x \in \mathbb{Z}, \forall y \in \mathbb{Z}, x + y < 0$" is false.

Problem 3 Suppose a, b are irrational and r is rational. Which of the following *must* be irrational? Provide either a proof or counterexample. You may use the fact that $\sqrt{2}$ is irrational; if you wish to use any other irrational numbers, you must justify that they are irrational.

(a) $a + r$
(b) $a + b$
(c) ar
(d) ab
(e) r^a
(f) (bonus) a^b

Problem 4 Prove that if n is a positive integer, then there is an integer x such that $n \leq x^2 \leq 2n$.