Discrete Mathematics

Midterm 2 Practice Exam

Name: _____

UNI: _____

Instructions.

- You have **75 minutes**.
- This exam is closed-book. No calculators or electronic devices.
- You may have up to one page of double-sided handwritten notes.
- There are 5 problems, worth 100 points in total.
- Write your solutions in the space provided. If you need more room, continue on the back of the page and say so clearly.

Question	Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
Total:	100	

Problem 1 (20 points)

For each of these questions, give an answer and a brief (1–2 sentence) explanation.

(a) Give a partition of the integers $\mathbb{Z}$ into 3 nonempty sets.

(b) Is $[0, 1)$ finite, countably infinite, or uncountable?

(c) How many ways can I distribute 30 oranges among 3 friends if each must get at least 5?

(d) Draw a 5-vertex graph with vertices of degree 2, 2, 3, 4, 4, or explain why it is not possible.

Problem 2 (20 points)

Twelve (distinct) teddy bears line up to buy tickets to see *Paddington 3*. Andrew and Bartholomew got in a fight recently and refuse to be next to each other. How many ways can the bears arrange themselves in a line?

Problem 3 (20 points)

Using counting in two ways, prove that for $m \leq k$,

$$\binom{m+n}{k} = \sum_{r=0}^{m} \binom{m}{r}\binom{n}{k-r}$$

Problem 4 (20 points)

Let $n \geq 6$, and consider K_n, the complete graph with vertices $\{1, 2, \cdots n\}$.

(a) How many length-5 *simple* paths are there in K_n?

(b) How many length-5 *simple cycles* are there in K_n that start (and end) at vertex 1?

(c) How many length-5 paths (not necessarily simple) are there in K_n?

Problem 5 (20 points)

Let G be a graph with n vertices. Prove that if every vertex has degree $\geq \frac{n}{2}$, then G is connected.

Hint: a graph is connected if every pair of vertices $u, v \in V$ have a path between them.