Productions for 3-Way π-Merges

a supplement to the paper

Embeddings of Cubic Halin Graphs: A Surface-by-Surface Inventory

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In the paper "Embeddings of Cubic Halin Graphs: A Surface-by-Surface Inventory", the cubic Halin graphs are represented as lying within a recursively defined family $\mathcal{F}$ of graphs. The family $\mathcal{F}$ has a single base graph $K_4 - e$ (which is its only non-Halin graph). A single operation called a π-merge is used to construct larger graphs in $\mathcal{F}$ from smaller graphs. The genus distributions of the graphs in $\mathcal{F}$ are partitioned into six non-zero partial distributions.

Accordingly, there are $36 = 6 \times 6$ productions for calculating the genus distributions of larger graphs in $\mathcal{F}$ from smaller graphs in $\mathcal{F}$. The six partials

$$dd' \quad dd'' \quad ds' \quad sd' \quad ss^1 \quad ss^2$$

and the operation π-merge are defined in §4 of the paper itself. Each of the 36 figures presented here illustrates one of these 36 productions.

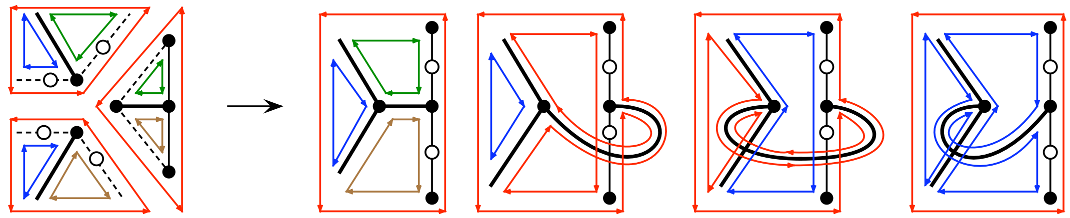

Figure 0.1: $dd'_i * dd'_j \longrightarrow dd'_{i+j} + 2dd''_{i+j+1} + ss^2_{i+j+1}$.

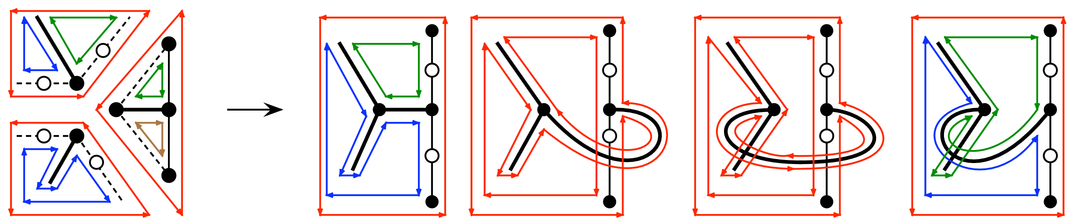

Figure 0.2: $dd'_i * dd''_j \longrightarrow 2dd'_{i+j} + 2ss^2_{i+j+1}$.

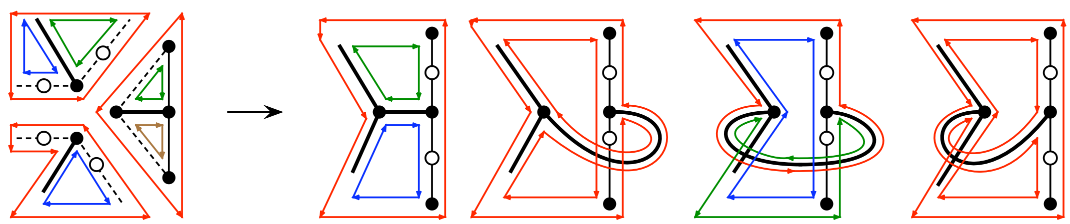

Figure 0.3: $dd'_i * ds'_j \longrightarrow 2dd'_{i+j} + 2ss^2_{i+j+1}$.

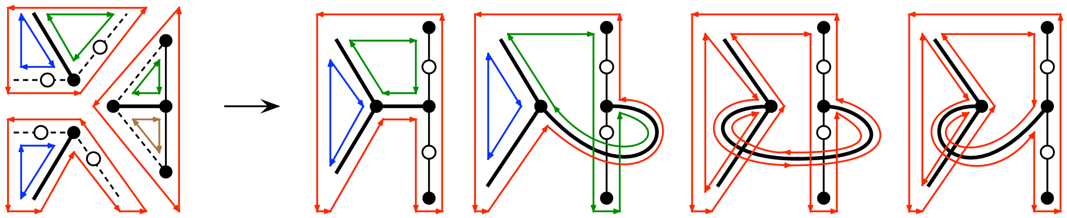

Figure 0.4: $dd'_i * sd'_j \longrightarrow 2sd'_{i+j} + 2ss^1_{i+j+1}$.

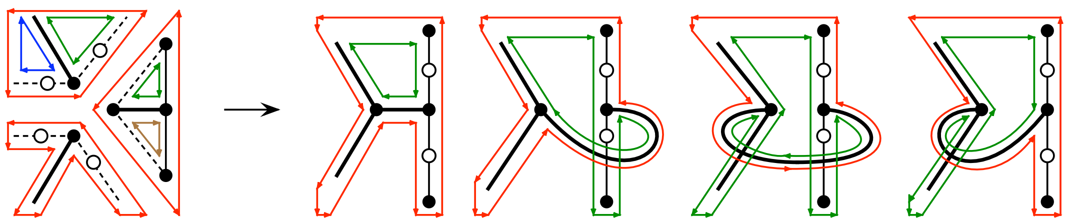

Figure 0.5: $dd'_i * ss^1_j \longrightarrow 4sd'_{i+j}$.

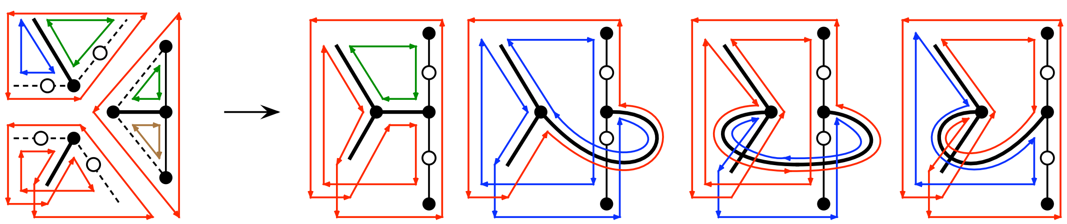

Figure 0.6: $dd'_i * ss^2_j \longrightarrow 2ds'_{i+j} + 2sd'_{i+j}$.

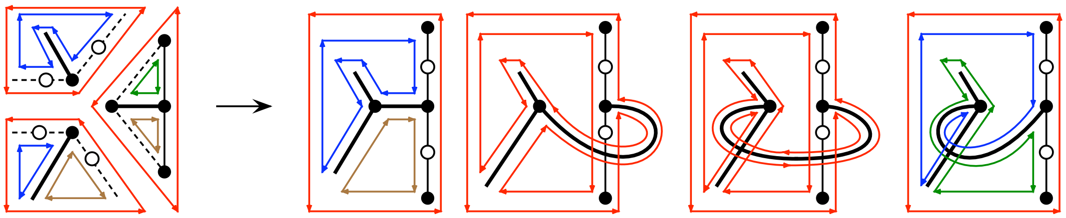

Figure 0.7: $dd''_i * dd'_j \longrightarrow 2dd'_{i+j} + 2ss^2_{i+j+1}$.

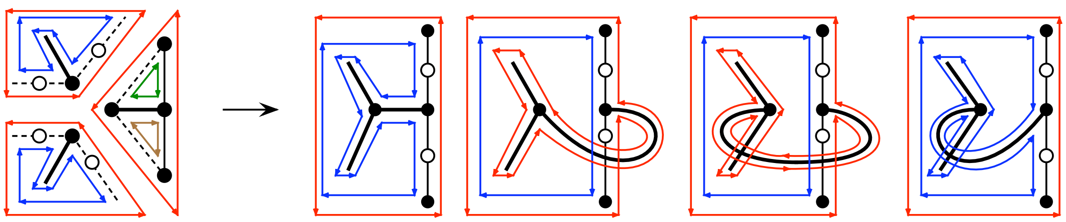

Figure 0.8: $dd''_i * dd''_j \longrightarrow 4dd''_{i+j}$.

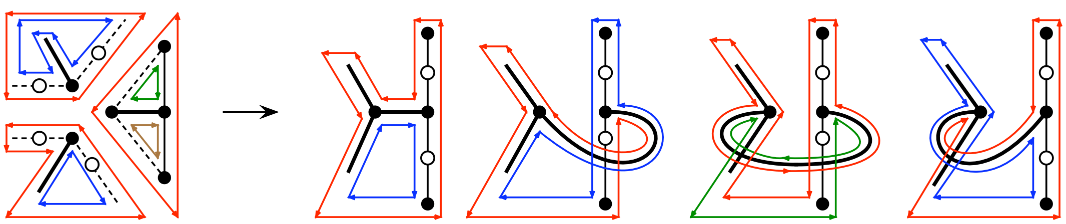

Figure 0.9: $dd''_i * ds'_j \longrightarrow 4ds'_{i+j}$.

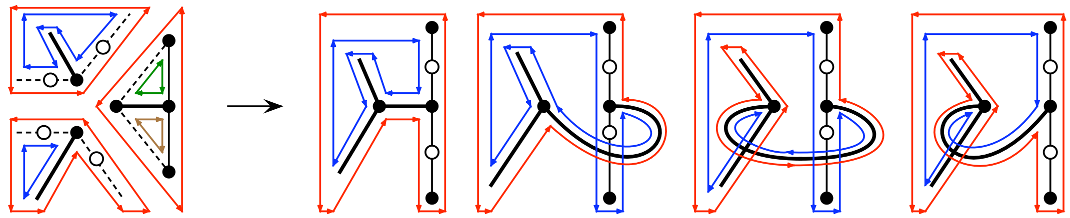

Figure 0.10: $dd''_i * sd'_j \longrightarrow 4sd'_{i+j}$.

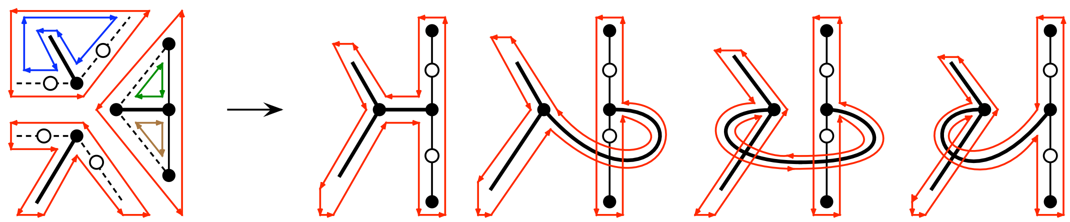

Figure 0.11: $dd''_i * ss^1_j \longrightarrow 4ss^1_{i+j}$.

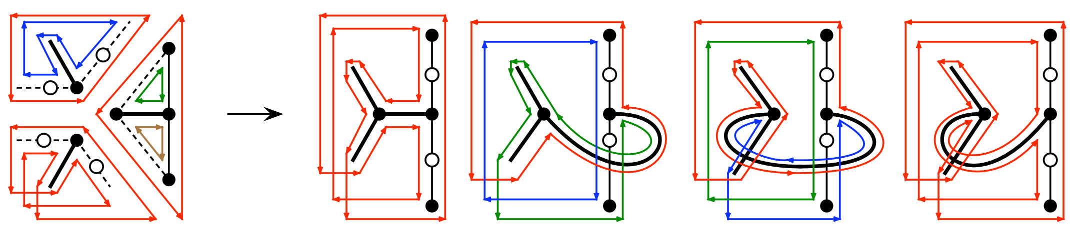

Figure 0.12: $dd''_i * ss^2_j \longrightarrow 2dd'_{i+j-1} + 2ss^2_{i+j}$.

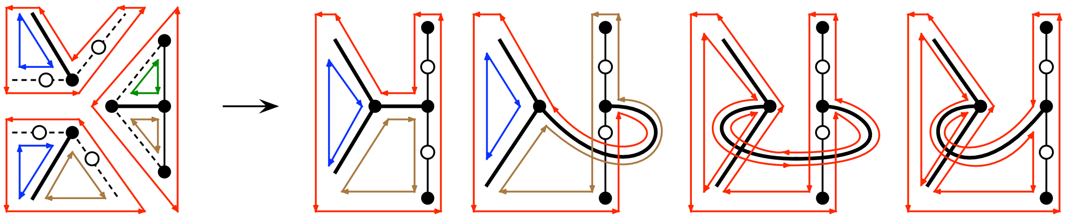

Figure 0.13: $ds'_i * dd'_j \longrightarrow 2ds'_{i+j} + 2ss^1_{i+j+1}$.

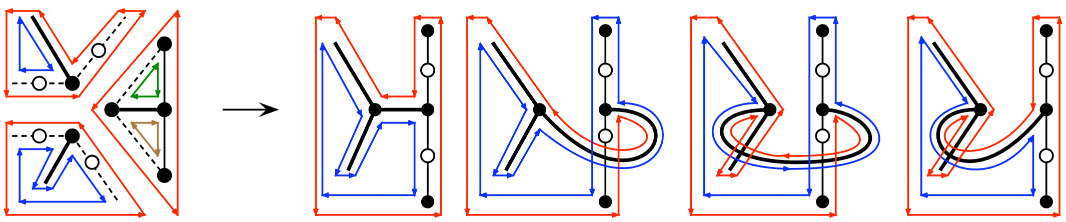

Figure 0.14: $ds'_i * dd''_j \longrightarrow 4ds'_{i+j}$.

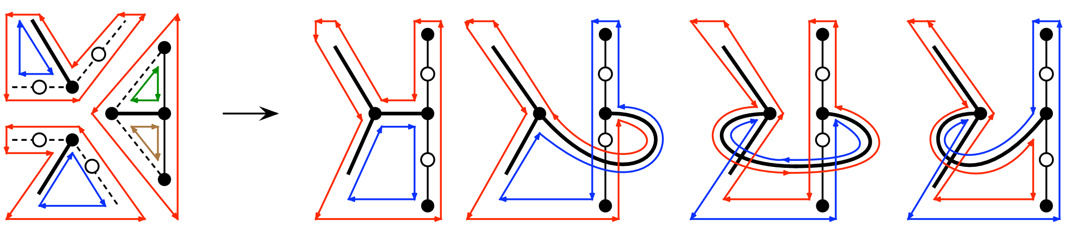

Figure 0.15: $ds'_i * ds'_j \longrightarrow 4ds'_{i+j}$.

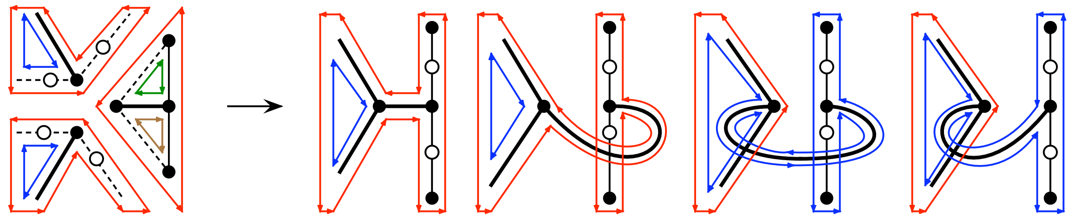

Figure 0.16: $ds'_i * sd'_j \longrightarrow 4ss^1_{i+j}$.

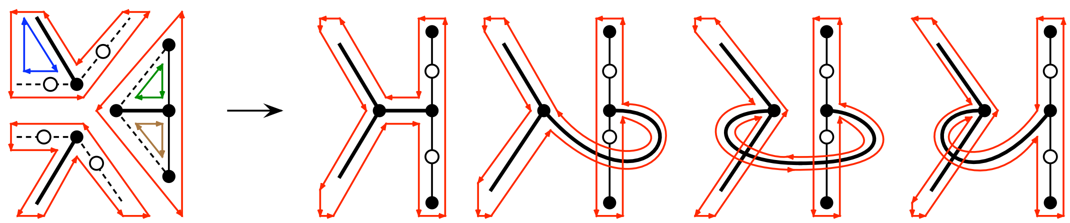

Figure 0.17: $ds'_i * ss^1_j \longrightarrow 4ss^1_{i+j}$.

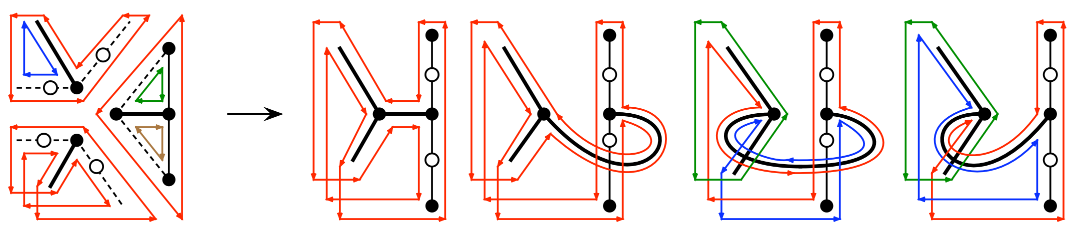

Figure 0.18: $ds'_i * ss^2_j \longrightarrow 2ds'_{i+j-1} + 2ss^1_{i+j}$.

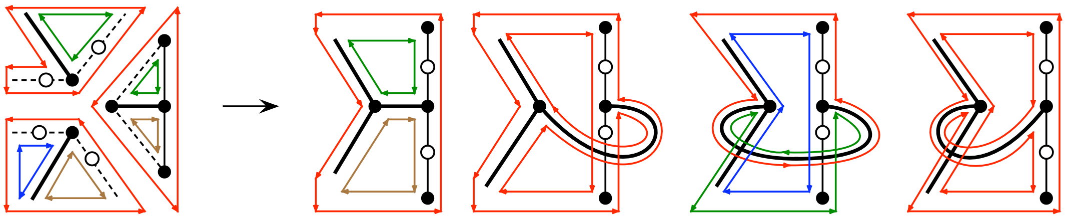

Figure 0.19: $sd'_i * dd'_j \longrightarrow 2dd'_{i+j} + 2ss^2_{i+j+1}$.

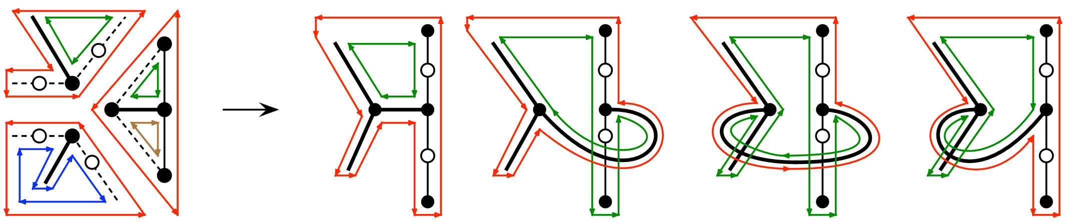

Figure 0.20: $sd'_i * dd''_j \longrightarrow 4sd'_{i+j}$.

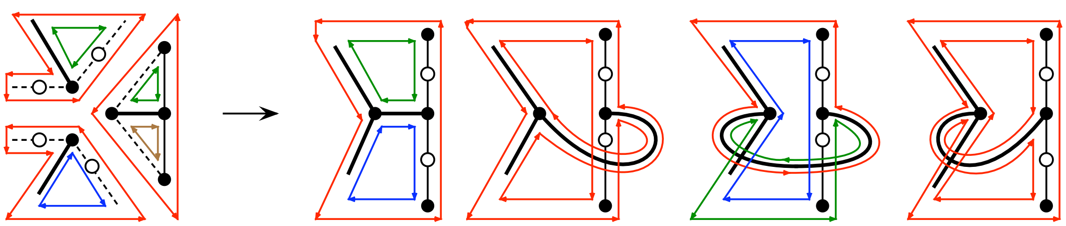

Figure 0.21: $sd'_i * ds'_j \longrightarrow 2dd'_{i+j-1} + 2ss^2_{i+j}$.

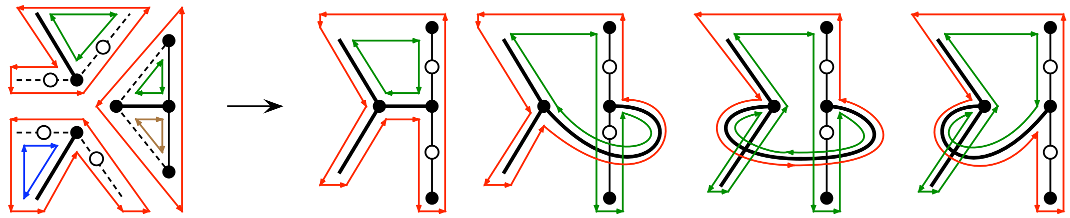

Figure 0.22: $sd'_i * sd'_j \longrightarrow 4sd'_{i+j}$.

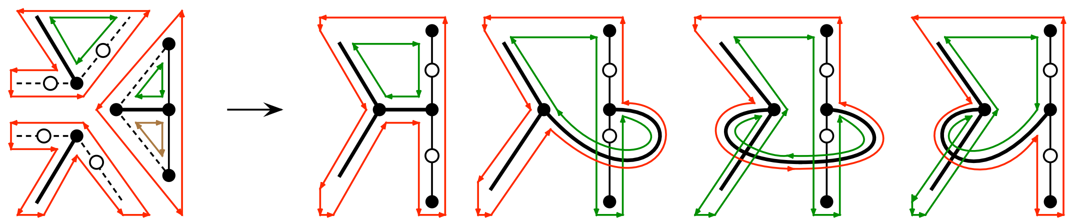

Figure 0.23: $sd'_i * ss^1_j \longrightarrow 4sd'_{i+j-1}$.

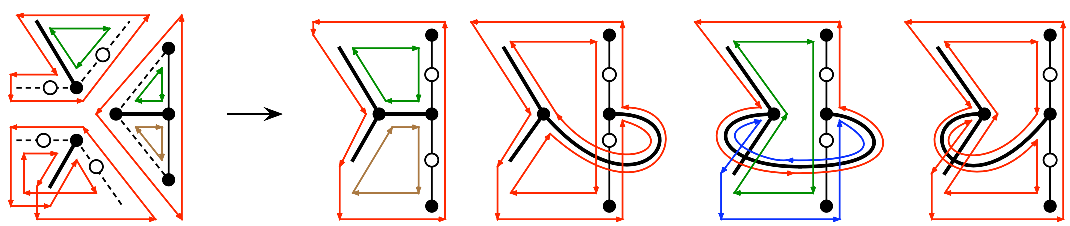

Figure 0.24: $sd'_i * ss^2_j \longrightarrow 2dd'_{i+j-1} + 2ss^2_{i+j}$.

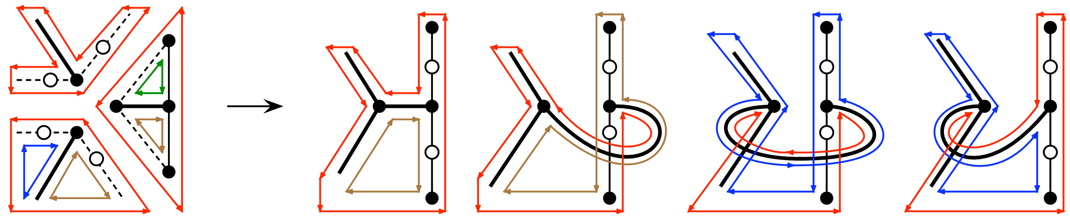

Figure 0.25: $ss_i^1 * dd'_j \longrightarrow 4ds'_{i+j}$.

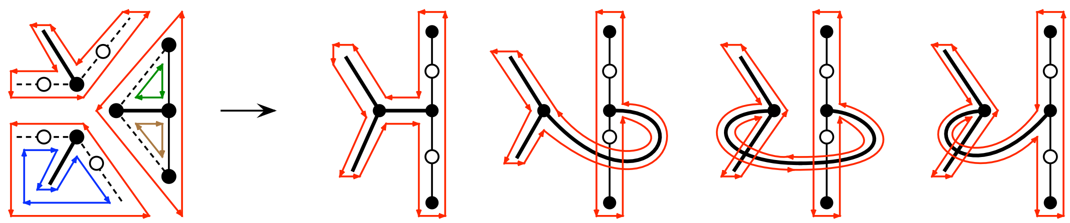

Figure 0.26: $ss_i^1 * dd''_j \longrightarrow 4ss^1_{i+j}$.

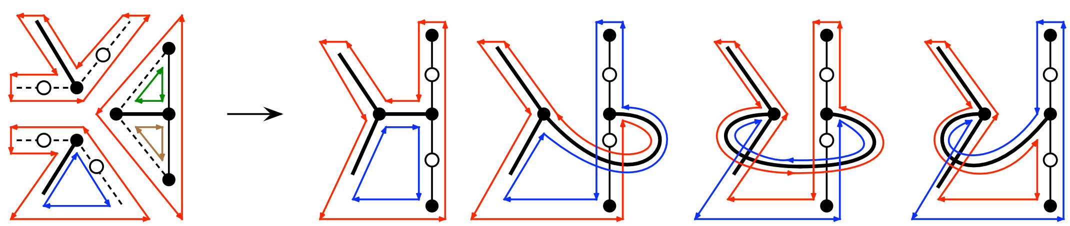

Figure 0.27: $ss_i^1 * ds'_j \longrightarrow 4ds'_{i+j-1}$.

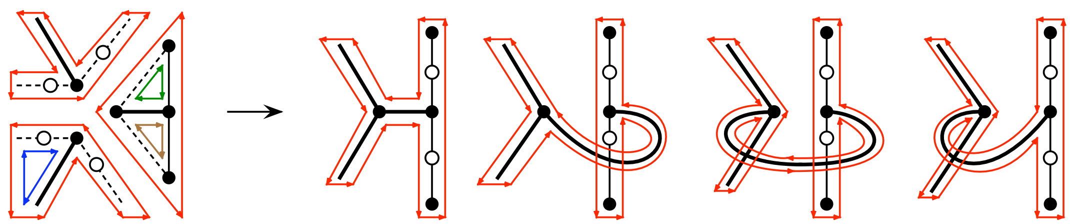

Figure 0.28: $ss^1_i * sd'_j \longrightarrow 4ss^1_{i+j}$.

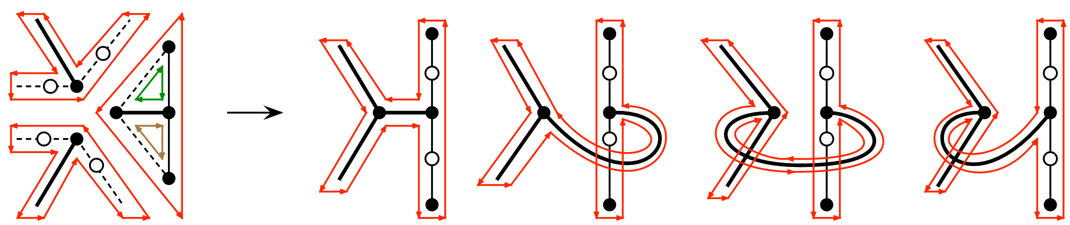

Figure 0.29: $ss^1_i * ss^1_j \longrightarrow 4ss^1_{i+j-1}$.

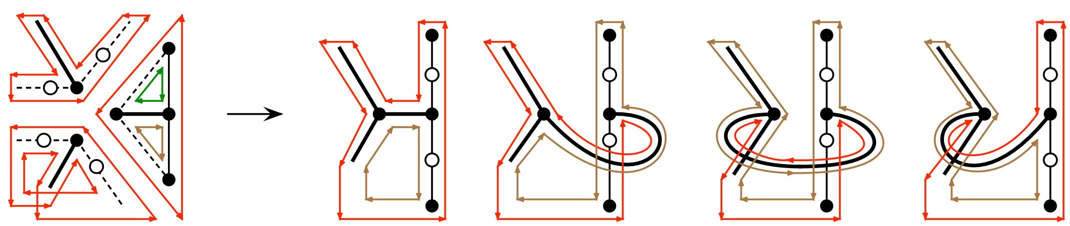

Figure 0.30: $ss^1_i * ss^2_j \longrightarrow 4ds'_{i+j-1}$.

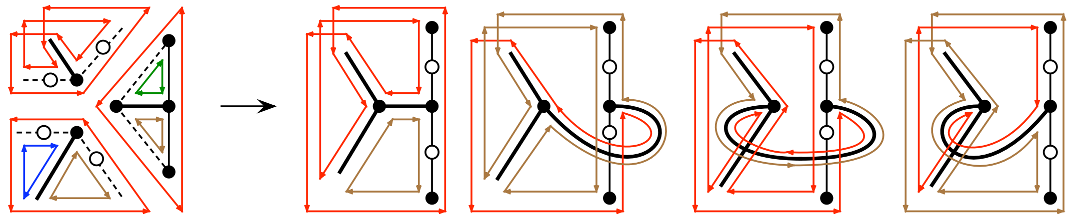

Figure 0.31: $ss_i^2 * dd_j' \longrightarrow 2ds_{i+j}' + 2sd_{i+j}'$.

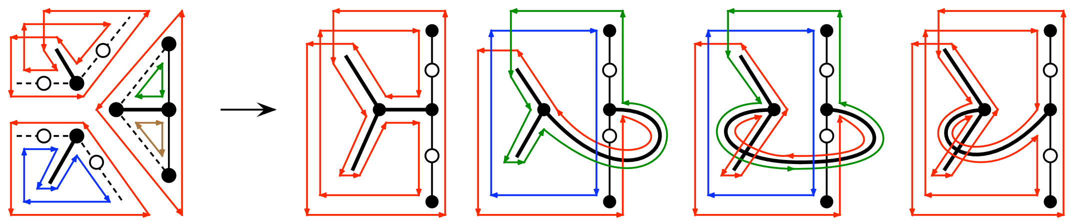

Figure 0.32: $ss_i^2 * dd_j'' \longrightarrow 2dd_{i+j-1}' + 2ss_{i+j}^2$.

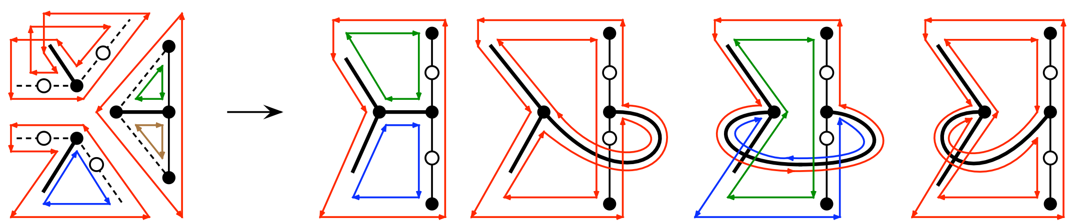

Figure 0.33: $ss_i^2 * ds_j' \longrightarrow 2dd_{i+j-1}' + 2ss_{i+j}^2$.

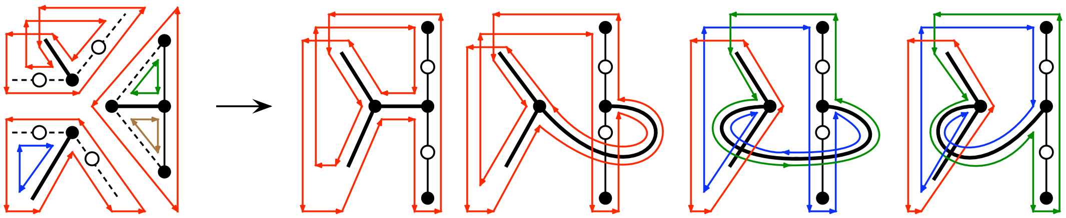

Figure 0.34: $ss_i^2 * sd'_j \longrightarrow 2sd'_{i+j-1} + 2ss^1_{i+j}$.

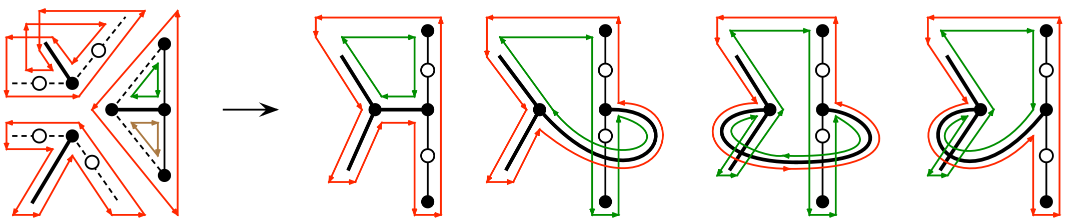

Figure 0.35: $ss_i^2 * ss_j^1 \longrightarrow 4sd'_{i+j-1}$.

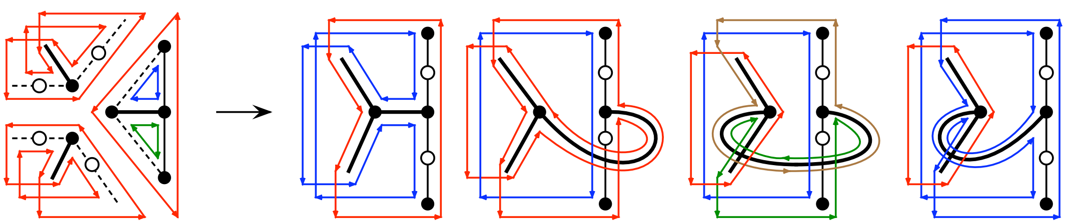

Figure 0.36: $ss_i^2 * ss_j^2 \longrightarrow 2dd''_{i+j-1} + dd'_{i+j-2} + ss^2_{i+j-1}$.