



CS1001

Lecture 23

Overview

- Incompleteness and the Halting Problem
- Methods in Artificial Intelligence

Reading

- Brookshear – Chapters 10 and 11

Functions

- Computer Science is concerned with implementing and evaluating mathematical functions
- There is now a practical concern – how to actually produce an output given an input
 - Functions can be classified as “computable” or “non-computable”

Turing Machines

- A theoretical device
 - Tape (memory)
 - A head (for reading/writing the tape)
 - Instructions for moving/reading/writing based on a current *state*
 - As powerful as any modern machine (in other words, it can compute everything a modern computer can compute)
 - <http://wap03.informatik.fh-wiesbaden.de/weber1/turing/tm.html>

Example

- The “successor” function
 - A function that takes an input and returns its value plus 1
 - $\text{Succ}(x) = x + 1$
 - $\text{Succ}(5)$ is equal to 6

Turing Computable

- Succ is Turing Computable because it can be computed using a turing machine
- How?
 - Start with the number (input) in binary on the tape
 - Go to the rightmost bit, if 0, change to 1 and halt
 - Otherwise, set to 0 and enter the “carry” state. Then, move backwards trying to find a 0...

Church-Turing Thesis

- “Turing-Computable” means the same thing as Algorithmically computable.
- In other words, if you can prove a function computable on a turing machine, it must also be computable by a definite algorithm
- The converse also holds

Computable vs Unknown

- Key Point: A non computable function is one where there *is* a definite correct answer *and* we have enough information to find it. However, actually getting the answer from the inputs is not feasible due to the complexity of the problem.
- This is *not* like asking “will the human race exist in 5,000 years” – we do not have enough information to answer that

Traveling Salesman Problem

- <http://www.math.princeton.edu/tsp/cpapp/us41.html>

The Halting Problem

- Given a program and inputs, is it possible to write *another* program to predict whether the initial program will terminate or loop for ever?
- No!

Public Key Cryptography
