Deadlock analysis for handoff events

• Verification allows one to find out
  – If any specific sequence of transitions/operations lead to any deadlock
  – If one specific state during handoff operation is attainable from any other state by following a specific sequence of transitions
  – Whether the coverability tree is reversible
    • The system comes back to the original state

• Methodology
  – Reachability analysis
  – Incidence Matrix-based equations
Petri Net for Matrix Equation Analysis

A sample Petri net
Incidence Matrix Analysis

\[ D^- = \begin{pmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \]

\[ D^+ = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \]

\[ D = \begin{pmatrix} 0 & -1 & -1 & 0 \\ 0 & +2 & +1 & -1 \\ 0 & 0 & -1 & +1 \end{pmatrix} \]
Matrix Analysis

\[ \mu' = \mu + x \cdot D \]

When the initial marking is \( \mu = (1,0,1,0) \)

transition \( t_3 \) is enabled

\[ \mu' = (1,0,1,0) + (0,0,1) \cdot \begin{pmatrix} 0 & -1 & -1 & 0 \\ 0 & +2 & +1 & -1 \\ 0 & 0 & -1 & +1 \end{pmatrix} \]

\[ = (1,0,1,0) + (0,0,-1,+1) \]

\[ = (1,0,0,1) \]

Enabling \( t_3 \) represents a sequence \( f(\sigma) = (0,0,1) \)
Matrix equation-based approach

Given a sequence $\sigma = t_3t_2t_3t_2t_1$ translates to a firing vector $f(\sigma) = (1,2,2)$, one can determine the marking $\mu'$ as

$$\mu' = (1,0,1,0) + (1,2,2).$$

$$\mu' = (1,3,0,0)$$

In order to determine if a marking $(1,8,0,1)$ is reachable from the marking $(1,0,1,0)$, it can have the equation

$$(1,8,0,1) = (1,0,1,0) + x \begin{pmatrix} 0 & -1 & -1 & 0 \\ 0 & +2 & +1 & -1 \\ 0 & 0 & -1 & +1 \end{pmatrix}.$$  

$$(0,8,-1,1) = x \begin{pmatrix} 0 & -1 & -1 & 0 \\ 0 & +2 & +1 & -1 \\ 0 & 0 & -1 & +1 \end{pmatrix}.$$  

Gives a solution $x = (0,4,5)$ that corresponds to the sequence $\sigma = t_3t_2t_3t_2t_3t_2t_3t_3$. Thus $\mu'$ is reachable from $\mu$ with this sequence of transitions.
Matrix equation

Given a sequence $\sigma = t_3t_2t_3t_2t_1$ translates to a firing vector $f(\sigma) = (1,2,2)$, one can determine the marking $\mu'$ as

$$\mu' = (1, 0,1,0) + (1,2,2) = (1,3,0,0)$$

In order to determine if a marking $(1,8,0,1)$ is reachable from the marking $(1,0,1,0)$, it can have the equation

$$(1,8,0,1) = (1,0,1,0) + x(0,8,-1,1) = x.$$

Gives a solution $x = (0,4,5)$ that corresponds to the sequence

$\sigma=t_3t_2t_3t_2t_3t_2t_3$. Thus $\mu'$ is reachable from $\mu$ with this sequence of transitions.
Reachability and Matrix analysis as applied to individual handoff operations

- Discovery
- Network Attachment
- Authentication
- Discovery
- Combination of any of these operations
Reachability and matrix analysis of handoff operation using Petri nets

Discovery
$M_0 = [1, 0, 0, 3, 4, 3, 0]^T$
Transition $t_1$ fires $\Rightarrow M_1 = [0, 1, 0, 1, 2, 1, 0]^T$
Transition t2 fires -> $M_2 = [0, 0, 1, 2, 1, 1, 0]^T$
Transition $t_3$ fires -> $M_3 = [0, 0, 1, 3, 4, 3, 1]^T$
Reachability Analysis

\[ M_0 = [0,1,1,3,4,3,0] \quad \text{Initial marking} \]
\[ M_1 = [0,1,0,1,2,1,0] \]
\[ M_2 = [0,0,1,2,1,1,0] \]
\[ M_3 = [0,0,1,3,4,3,1] \]

M0 can reach the state M3 by following a sequence of transition t1t2t3
Incidence Matrix Analysis

Input Matrix $D^-$ =
\[
\begin{pmatrix}
1 & 0 & 0 & 2 & 2 & 2 & 0 \\
0 & 1 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0
\end{pmatrix}
\]

Output matrix $D^+$ =
\[
\begin{pmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 3 & 2 & 1
\end{pmatrix}
\]

Incidence matrix $D = D^+ - D^-$ =
\[
\begin{pmatrix}
-1 & 1 & 0 & -2 & -2 & -2 & 0 \\
0 & -1 & 1 & 1 & -1 & 0 & 0 \\
0 & 0 & 0 & -1 & 3 & 2 & 1
\end{pmatrix}
\]
Matrix equation-based approach

\[ \mu' = \mu + x.D \]

Given a sequence \( \sigma = t_1t_2t_3 \) translates to a firing vector \( f(\sigma) = (1,1,1) \), one can determine the marking \( \mu' \) as

\[ \mu' = (1, 0,0,3,4,3,0) + (1,1,1). \begin{pmatrix} -1 & 1 & 0 & -2 & -2 & -2 & 0 \\ 0 & -1 & 1 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 3 & 2 & 1 \end{pmatrix} \]

\[ \mu' = (1,0,0,3,4,3,0) + (-1,0,0,0,0,0,1) \]
\[ \mu' = (0,0,0,3,4,31) \] final marking

Thus, \( \mu' \) is reachable from the initial marking with a sequence of transition \( t_1,t_2,t_3 \) (\( t_1, t_2 \) and \( t_3 \) are enabled) that corresponds to \( (1,1,1) \)
Reachability and matrix analysis of handoff operation using Petri nets

(Network Attachment)
Initial Marking: \( M_0 = [1, 0, 0, 3, 4, 3, 0] \)
t1 fires Marking: $M_1 = [0,1,0,2,3,2,0]$
t2 fires Marking: \( M_2 = [0,0,1,2,3,1,0] \)
t3 fires Marking: $M_3 = [0,0,1,2,3,1,0]$
Reachability Analysis

\[ M_0 = [1,0,0,3,4,3,0] \]  \text{Initial marking}

\[ M_1 = [0,1,0,2,3,2,0] \]

\[ M_2 = [0,0,1,2,3,1,0] \]

\[ M_3 = [0,0,1,2,3,1,0] \]
Incidence Matrix Analysis

Input Matrix $D^- = \begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$

Output matrix $D^+=$ \[ \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 2 & 1 \end{pmatrix} \]

Incidence matrix $D = D^+ - D^- = \begin{pmatrix} -1 & 1 & 0 & -1 & -1 & -1 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & -1 & 1 & 1 & 2 & 1 \end{pmatrix}$
Matrix equation-based approach

\[ \mu' = \mu + x.D \]

Given a sequence \( \sigma = t_1t_2t_3 \) translates to a firing vector \( f(\sigma) = (1,1,1) \), one can determine the marking \( \mu' \) as

\[ \mu' = (1, 0, 0, 3, 4, 3, 0) + (1, 1, 1). \]

\[ \begin{pmatrix}
-1 & 1 & 0 & -1 & -1 & -1 & 0 \\
0 & -1 & 1 & 0 & 0 & -1 & 0 \\
0 & 0 & -1 & 1 & 1 & 2 & 1
\end{pmatrix} \]

\[ \mu' = (1, 0, 0, 3, 4, 3, 0) + (-1, 0, 0, 0, 0, 0, 1) \]

\[ \mu' = (0, 0, 0, 3, 4, 3, 1) \text{ final marking} \]

Thus, \( \mu' \) is reachable from the initial marking with a sequence of transition \( t_1, t_2, t_3 \) that corresponds to \( (1,1,1) \)
Reachability and Matrix analysis of handoff operation using Petri nets

Authentication
Authentication Process

Mobile Authenticated

(WEP Key)

Open Auth

EAP

P5

(Resource Memory PM)

(Resource Bandwidth PB)

(Resource Processing Power PP)
M₀ = [1, 0, 3, 4, 3, 0]ᵀ  Initial Marking

Authentication Process
t1 fires $M_1 = [0, 1, 1, 3, 1, 0]^T$
t2 fires $M2 = [0, 0, 3, 4, 3, 1]^T$
Reachability Analysis

\[ M_0 = [1003430]^T \]

- t1 fires

\[ M_1 = [0,1,1,3,1,0]^T \]

- t2 fires

\[ M_2 = [0,0,3,4,3,1]^T \]
Incidence Matrix Analysis

Input Matrix $D^-$ =

\[
\begin{pmatrix}
1 & 0 & 0 & 1 & 2 & 0 \\
0 & 1 & 2 & 1 & 1 & 0 \\
\end{pmatrix}
\]

Output matrix $D^+$ =

\[
\begin{pmatrix}
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 2 & 2 & 3 & 1 \\
\end{pmatrix}
\]

Incidence matrix $D = D^+ - D^- =

\[
\begin{pmatrix}
-1 & 1 & 0 & -1 & -2 & 0 \\
0 & -1 & 0 & 1 & 2 & 1 \\
\end{pmatrix}
\]
Matrix equation-based approach

\[ \mu' = \mu + x.D \]

Given a sequence \( \sigma = t_1t_2t_3 \) translates to a firing vector \( f(\sigma) = (1,1,1) \), one can determine the marking \( \mu' \) as

\[ \mu' = (1, 0,3,4,3,0) + (1,1). \begin{pmatrix} -1 & 1 & 0 & -1 & -2 & 0 \\ 0 & -1 & 0 & 1 & 2 & 1 \end{pmatrix} \]

\[ \mu' = (1,0,3,4,3,0) + (-1,0,0,0,0,1) \]
\[ \mu' = (0,0,3,4,31) \]

Thus, \( \mu' \) is reachable from the initial marking with a sequence of transition \( t_1,t_2,t_3 \) that corresponds to \( (1,1,1) \)
Reachability and matrix analysis of handoff operation using Petri nets

Configuration
$M_0 = [1003430]^T = \text{Initial Marking}$

Configuration Process
T1 fires : $M1 = [0102120]^T$
t2 fires: $M2 = [0010220]^T$
t3 fires : M3= [0003431]^T
Reachability Analysis

\[
M_0 = [1003430]^T
\]

\[
\text{t1 fires}
\]

\[
M_1 = [0102120]^T
\]

\[
\text{t2 fires}
\]

\[
M_2 = [0010220]^T
\]

\[
\text{t3 fires}
\]

\[
M_3 = [0003431]^T
\]
Incidence Matrix Analysis

Input Matrix $D^{-} =$
\[
\begin{pmatrix}
1 & 0 & 0 & 1 & 3 & 1 & 0 \\
0 & 1 & 0 & 2 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 3 & 2 & 1 & 1
\end{pmatrix}
\]

Output matrix $D^{+}$ =
\[
\begin{pmatrix}
1 & 0 & 0 & 1 & 3 & 1 & 0 \\
0 & 1 & 0 & 2 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 3 & 2 & 1 & 1
\end{pmatrix}
\]

Incidence matrix $D = D^{+} - D^{-} =$
\[
\begin{pmatrix}
-1 & 1 & 0 & -1 & -3 & -1 & 0 \\
0 & -1 & 1 & -2 & 1 & 0 & 0 \\
0 & 0 & -1 & 3 & 2 & 1 & 1
\end{pmatrix}
\]
Matrix equation-based approach

\[ \mu' = \mu + x \cdot D \]

Given a sequence \( \sigma = t_1 t_2 t_3 \) translates to a firing vector \( f(\sigma) = (1, 1, 1) \), one can determine the marking \( \mu' \) as

\[ \mu' = (1, 0, 0, 3, 4, 3, 0) + (1, 1, 1) \]

\[ \mu' = (1, 0, 0, 3, 4, 3, 0) + (-1, 0, 0, 0, 0, 0, 1) \]

\[ \mu' = (0, 0, 0, 3, 4, 31) \]

Thus, \( \mu' \) is reachable from the initial marking with a sequence of transition \( t_1, t_2, t_3 \) that corresponds to \( (1, 1, 1) \)
Putting together 4 operations