Interactive learning via reductions

Daniel Hsu
Columbia University

May 9, 2019

Simons Symposium on New Directions in Theoretical Machine Learning

Interactive learning via reductions

("How can we build on the recent success in supervised learning?")

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Interactive learning: Contextual bandits

Website operator

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Loop:

1. User visits website with profile, browsing history . . .
2. Choose content to display on website.
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Goal: choose content that yield desired user behavior.

Interactive learning: Active learning

E-mail service provider

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1. Receive e-mail messages for users (spam or not).
2. Ask users to provide labels for some (borderline) messages.
3. Improve spam filter using newly labeled messages.

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Goal: maximize accuracy of spam filter, minimize queries to users.

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Efficient solutions to **exploration/exploitation dilemma** via reductions to supervised learning

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Rest of this talk:

1. Reductions for contextual bandits
2. Some challenges with this approach
   (an excuse to talk about generalization?)

1. Contextual bandit learning

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Contextual: use features x_t to choose good actions a_t.

Bandit: $r_t(a)$ for $a \neq a_t$ is not observed.

(Non-bandit setting: whole reward vector $\boldsymbol{r}_t \in [0,1]^{\mathcal{A}}$ is observed.)

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3. Selection bias, especially while *exploiting*.

Learning objective

Regret (*i.e.*, relative performance) to a policy class Π:

$$\underbrace{\max_{\pi \in \Pi} \frac{1}{T} \sum_{t=1}^{T} r_t(\pi(x_t))}_{\text{average reward of best policy}} \quad - \quad \underbrace{\frac{1}{T} \sum_{t=1}^{T} r_t(a_t)}_{\text{average reward of learner}}$$

Strong benchmark when Π has a policy w/ high expected reward.

Goal: regret $\to 0$ as fast as possible as $T \to \infty$.

Contextual bandits via reduction to supervised learning

Let $K := |\mathcal{A}|$ and $N := |\Pi|$.

Algorithm that operates via reduction to supervised learning
(Agarwal, H., Kale, Langford, Li, & Schapire, 2014).

- Regret bound: $\tilde{O}\left(\sqrt{\frac{K \log N}{T}}\right)$.
  Near optimal statistical performance
- # calls to supervised learner for Π: $\tilde{O}\left(\sqrt{\frac{TK}{\log N}}\right)$.
  Uses supervised learner less than once per round

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$$(x_1, \boldsymbol{r}_1), \ldots, (x_t, \boldsymbol{r}_t) \in \mathcal{X} \times \mathbb{R}^{\mathcal{A}}.$$

context	$\longrightarrow$	features
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- Can often exploit structure of Π to get tractable algorithms. **Abstraction for supervised learning**: arg max oracle (AMO)

$$\mathrm{AMO}\left(\{(x_i, \boldsymbol{r}_i)\}_{i=1}^{t}\right) := \operatorname*{arg\,max}_{\pi \in \Pi} \sum_{i=1}^{t} r_i(\pi(x_i)) .$$

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In bandit setting: *use randomization + importance weighting*.
Draw $a_t \sim \boldsymbol{P}_t$ for some pre-specified prob. dist. $\boldsymbol{P}_t$.

Inverse propensity weighting (Horvitz & Thompson, 1952)

Importance-weighted estimate of reward from round t:

$$\forall a \in \mathcal{A} \,.\quad \hat{r}_t(a) \;:=\; \begin{cases} \dfrac{r_t(a_t)}{P_t(a)} & \text{if } a = a_t \,, \\ 0 & \text{otherwise} \,. \end{cases}$$

Estimate avg. reward of policy: $\widehat{\mathrm{Rew}}_t(\pi) := \frac{1}{t}\sum_{i=1}^{t} \hat{r}_i(\pi(x_i))$.

How should we choose action distribution P_t?

Hedging over policies

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Policy distribution: $\boldsymbol{Q} = (Q(\pi) : \pi \in \Pi)$
probability dist. over policies π in the policy class Π

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$$\underbrace{(\boldsymbol{Q}, x)}_{\text{(policy distribution, context)}} \quad \longmapsto \quad \underbrace{\boldsymbol{P}}_{\text{action distribution}}$$

Pick initial distribution $\boldsymbol{Q}_1$ over policies Π.
for round $t = 1, 2, \ldots$ **do**
    Nature draws $(x_t, \boldsymbol{r}_t)$ from dist. $\mathcal{D}$ over $\mathcal{X} \times [0,1]^{\mathcal{A}}$.
    Observe context x_t.
    Compute distribution $\boldsymbol{P}_t$ over $\mathcal{A}$ (using $\boldsymbol{Q}_t$ and x_t).
    Pick action $a_t \sim \boldsymbol{P}_t$.
    Collect reward $r_t(a_t)$.
    Compute new distribution $\boldsymbol{Q}_{t+1}$ over policies Π.
end for

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Theorem: Using feasible $\boldsymbol{Q}_t$ in round $t \Rightarrow$ near-optimal regret.

$\star$ Can implement efficient "coordinate descent" solver via AMO.

Implementation via AMO

Finding "low variance" constraint violation for $\boldsymbol{Q}$:

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1. Create fictitious rewards for each $i = 1, 2, \ldots, t$:

$$\widetilde{r}_i(a) \;:=\; K \cdot \frac{\hat{r}_i(a)}{\sqrt{\frac{K \log N}{t}}} + \frac{1}{Q(a|x_i)} \quad \forall a \in \mathcal{A}\,.$$

2. Obtain $\widetilde{\pi} := \mathrm{AMO}\big(\{(x_i, \widetilde{\boldsymbol{r}}_i)\}_{i=1}^t\big)$.

   Fact: $\widetilde{\mathrm{Rew}}_t(\widetilde{\pi}) >$ threshold iff $\widetilde{\pi}$'s constraint is violated.

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So what is the catch?

2. Problems

Major impediments with current reductions

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- **Similar issue with active learning**: generalization bounds are crucially used to measure prediction "confidence".

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  E.g., under what circumstances will the norm small?

Some surprising behavior

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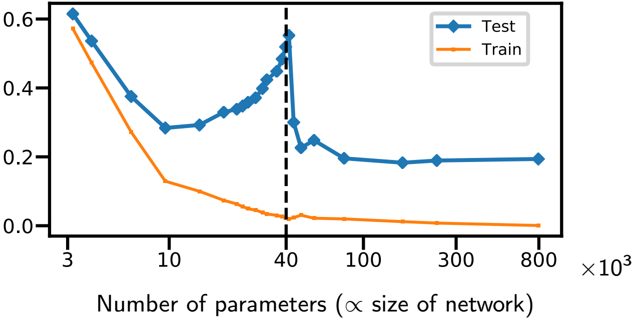

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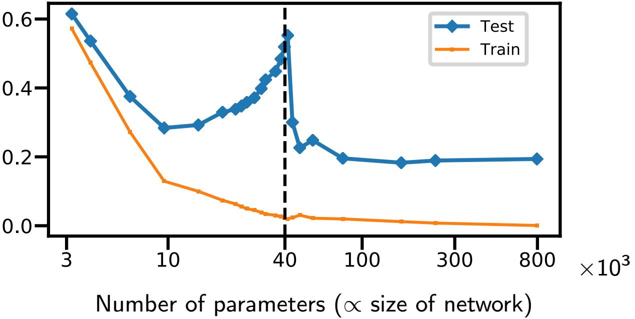

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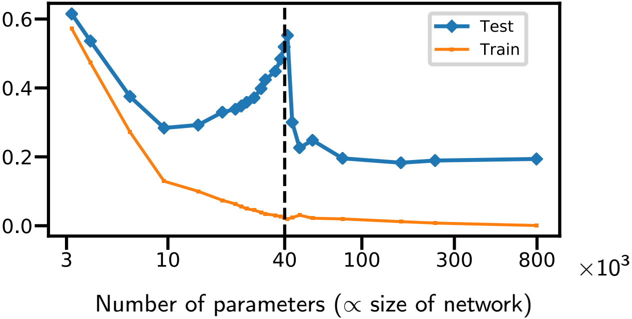

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Why do we observe good performance even when "overfitted"?

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  1. Simplicial interpolation (plausibly similar to ReLU networks)

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Would be great to have such results for interpolating neural nets, or even kernel machines.

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- **However**:
  Existing reductions crucially rely on generalization bounds.
  - Perhaps consequence of statistical learning framework . . .
  - Need better understanding of function classes we want to use (e.g., “practical” neural nets)

Concluding remarks

How to manage exploration for interactive learning?

- Reductions provide way to use advances in supervised learning to do better interactive learning.
- **However**:
  Existing reductions crucially rely on generalization bounds.
  - Perhaps consequence of statistical learning framework ...
  - Need better understanding of function classes we want to use (e.g., "practical" neural nets)

Thanks!

Simons Institute for the Theory of Computing

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3. Extra

(Sub-optimal) alternative

Explore-then-exploit:

1. Pick uniformly random actions in first τ rounds.
2. Obtain $\hat{\pi} := \mathrm{AMO}(\{(x_i, \hat{\boldsymbol{r}}_i\}))_{i=1}^{\tau}$.
3. Use $\hat{\pi}$ in remaining $T - \tau$ rounds.

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Other alternatives: replace bounds with resampling methods (e.g., permutation tests, bootstrap). Can these be made optimal?

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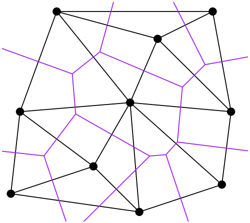

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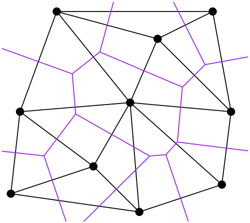

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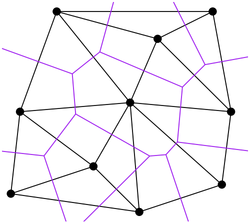

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- Define $\hat{\eta}(x)$ on each simplex by affine interp. of vertices' labels
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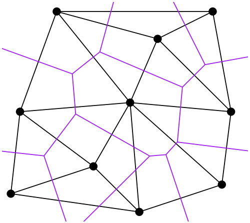
- For classification, let $\hat{f}$ be plug-in classifier via $\hat{\eta}$.



Comparison to nearest neighbor

Restrict attention to a single simplex, with vertices $x_1, \ldots, x_{d+1}$.

- Suppose $\Pr(y = 1 \mid x) < 1/2$ for all points in the simplex
- Suppose training data has

$$y_1 = \cdots = y_d = 0$$

but $y_{d+1} = 1$ (due to noise, say).