Reducing contextual bandits to supervised learning

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Based on joint work with A. Agarwal, S. Kale, J. Langford, L. Li, and R. Schapire
Learning to interact: example #1

Practicing physician
Learning to interact: example #1

Practicing physician
Loop:
1. Patient arrives with symptoms, medical history, genome...
Learning to interact: example #1

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3. Observe impact on patient’s health (e.g., improves, worsens).
Learning to interact: example #1

Practicing physician
Loop:
1. Patient arrives with symptoms, medical history, genome . . .
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Goal: prescribe treatments that yield good health outcomes.
Learning to interact: example #2

Website operator
Learning to interact: example #2

Website operator
Loop:
1. User visits website with profile, browsing history...
Learning to interact: example #2

Website operator

Loop:

1. User visits website with profile, browsing history . . .

2. Choose content to display on website.
Learning to interact: example #2

**Website operator**

Loop:

1. User visits website with profile, browsing history . . .
2. Choose content to display on website.
3. Observe user reaction to content (e.g., click, “like”).
Learning to interact: example #2

Website operator
Loop:
1. User visits website with profile, browsing history . . .
2. Choose content to display on website.
3. Observe user reaction to content (e.g., click, “like”).

Goal: choose content that yield desired user behavior.
Contextual bandit problem

For $t = 1, 2, \ldots, T$:
Contextual bandit problem

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1. Observe context $x_t \in \mathcal{X}$. [e.g., user profile, search query]
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For $t = 1, 2, \ldots, T$:

0. Nature draws $(x_t, r_t)$ from dist. $D$ over $\mathcal{X} \times [0, 1]^A$.
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Task: choose \( a_t \)'s that yield high expected reward (w.r.t. \( \mathcal{D} \)).
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**Task:** choose $a_t$’s that yield high expected reward (w.r.t. $D$).

**Contextual:** use features $x_t$ to choose good actions $a_t$.

**Bandit:** $r_t(a)$ for $a \neq a_t$ is not observed.

(Non-bandit setting: whole reward vector $r_t \in [0, 1]^A$ is observed.)
Challenges

1. Exploration vs. exploitation
   ▶ Use what you've already learned (exploit), but also learn about actions that could be good (explore).
   ▶ Must balance to get good statistical performance.

2. Must use context
   ▶ Want to do as well as the best policy (i.e., decision rule) $\pi$: context $x$ $\mapsto$ action $a$ from some policy class $\Pi$ (a set of decision rules).
   ▶ Computationally constrained with large $\Pi$ (e.g., all decision trees).

3. Selection bias, especially while exploiting.
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Learning objective

Regret (i.e., relative performance) to a policy class \( \Pi \):

\[
\max_{\pi \in \Pi} \frac{1}{T} \sum_{t=1}^{T} r_t(\pi(x_t)) - \frac{1}{T} \sum_{t=1}^{T} r_t(a_t)
\]

average reward of best policy - average reward of learner

Strong benchmark if \( \Pi \) contains a policy with high expected reward!

Goal: regret \( \rightarrow 0 \) as fast as possible as \( T \rightarrow \infty \).
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**Goal**: regret $\to 0$ as fast as possible as $T \to \infty$. 
Our result

New fast and simple algorithm for contextual bandits.
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- Operates via reduction to supervised learning (with computationally-efficient reduction).
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New fast and simple algorithm for contextual bandits.

- Operates via reduction to supervised learning (with computationally-efficient reduction).
- Statistically (near) optimal regret bound.
Need for exploration

No-exploration approach:

1. Using historical data, learn a "reward predictor" for each action $a \in A$ based on context $x \in X$: $\hat{r}(a|x)$.
2. Then deploy policy $\hat{\pi}$, given by $\hat{\pi}(x) := \text{argmax}_{a \in A} \hat{r}(a|x)$, and collect more data.

Suffers from selection bias.
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Example: two contexts \{X, Y\}, two actions \{A, B\}.

Suppose initial policy says \(\hat{\pi}(X) = A\) and \(\hat{\pi}(Y) = B\).
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True rewards:

| \( X \) | 0.7 | 1.0 |
| \( Y \) | 0.3 | 0.1 |

Never try action \( B \) in context \( X \).
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Dealing with policies

Feedback in round $t$: reward of chosen action $r_t(a_t)$.

- Tells us about policies $\pi \in \Pi$ s.t. $\pi(x_t) = a_t$.
- Not informative about other policies!
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Statistically optimal regret bound $O(\sqrt{K \log N T})$ for $K = |A|$ actions and $N = |\Pi|$ policies after $T$ rounds.

Explicit bookkeeping is computationally intractable for large $N$. But perhaps policy class $\Pi$ has some structure...
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But perhaps policy class $\Pi$ has some structure ...
Hypothetical “full-information” setting

If we observed rewards for all actions...
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- Like supervised learning, have labeled data after $t$ rounds:

$$
(x_1, \rho_1), \ldots, (x_t, \rho_t) \in X \times \mathbb{R}^A.
$$
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Can often exploit structure of \( \Pi \) to get tractable algorithms.

Abstraction: argmax oracle (AMO): \[\text{AMO}\left(\{(x_i, \rho_i)\}_{i=1}^t\right) := \text{argmax}_{\pi \in \Pi} t \sum_{i=1}^t \rho_i(\pi(x_i)).\]

Can't directly use this in bandit setting.

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Hypothetical “full-information” setting

If we observed rewards for all actions . . .

- Like supervised learning, have labeled data after \( t \) rounds:

\[
(x_1, \rho_1), \ldots, (x_t, \rho_t) \in \mathcal{X} \times \mathbb{R}^A.
\]

| context | → | features |
| actions | → | classes |
| rewards | → | – costs |
| policy | → | classifier |

- Can often exploit structure of \( \Pi \) to get tractable algorithms.

**Abstraction:** \( \arg \max \) oracle (AMO)

\[
\text{AMO}\left( \{(x_i, \rho_i)\}_{i=1}^t \right) := \arg \max_{\pi \in \Pi} \sum_{i=1}^t \rho_i(\pi(x_i)).
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\]

Can’t directly use this in bandit setting.
Using AMO with some exploration

Explore-then-exploit

1. In first $\tau$ rounds, choose $a_t \in A$ u.a.r. to get unbiased estimates $\hat{r}_t$ of $r_t$ for all $t \leq \tau$.
2. Get $\hat{\pi} := \text{AMO}(\{(x_t, \hat{r}_t)\}_{t=1}^{\tau})$.
3. Henceforth use $a_t := \hat{\pi}(x_t)$, for $t = \tau + 1, \tau + 2, \ldots, T$.

Regret bound with best $\tau$: $\sim T - 1/3$ (sub-optimal).

(Dependencies on $|A|$ and $|\Pi|$ hidden.)
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(Dependencies on $|A|$ and $|\Pi|$ hidden.)
Previous contextual bandit algorithms


Optimal regret, but explicitly enumerates $\Pi$.

Greedy (Langford & Zhang, NIPS 2007)

Sub-optimal regret, but one call to AMO.

Monster (Dudik, Hsu, Kale, Karampatziakis, Langford, Reyzin, & Zhang, UAI 2011)

Near optimal regret, but $O(T^6)$ calls to AMO.
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Our result

Let $K := |\mathcal{A}|$ and $N := |\Pi|$.

Our result: a new, fast and simple algorithm.

- Regret bound: $\tilde{O}\left(\sqrt{\frac{K \log N}{T}}\right)$.
  Near optimal.

- # calls to AMO: $\tilde{O}\left(\sqrt{\frac{TK}{\log N}}\right)$.
  Less than once per round!
Rest of the talk

Components of the new algorithm:

2. Efficient algorithm for balancing exploration/exploitation.
3. Additional tricks: warm-start and epoch structure.
1. Classical tricks
What would’ve happened if I had done X?

For $t = 1, 2, \ldots, T$:

0. Nature draws $(x_t, r_t)$ from dist. $\mathcal{D}$ over $\mathcal{X} \times [0, 1]^A$.
1. Observe context $x_t \in \mathcal{X}$. [e.g., user profile, search query]
2. Choose action $a_t \in A$. [e.g., ad to display]
3. Collect reward $r_t(a_t) \in [0, 1]$. [e.g., 1 if click, 0 otherwise]
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Q: How do I learn about $r_t(a)$ for actions $a$ I don’t actually take?
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Q: How do I learn about \( r_t(a) \) for actions \( a \) I don’t actually take?

A: Randomize. Draw \( a_t \sim p_t \) for some pre-specified prob. dist. \( p_t \).
Inverse propensity weighting (Horvitz & Thompson, JASA 1952)

Importance-weighted estimate of reward from round $t$:

$$\forall a \in A. \quad \hat{r}_t(a) := \frac{r_t(a_t) \cdot 1\{a = a_t\}}{p_t(a_t)}$$
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How should we choose the $p_t$?
Hedging over policies

Get action distributions via policy distributions.

\[(Q, x) \rightarrow p\]

(policy distribution, context) \quad \text{action distribution}
Hedging over policies

Get action distributions via policy distributions.

\[(Q, x) \rightarrow p\]

(policy distribution, context) \rightarrow action distribution

**Policy distribution:** \( Q = (Q(\pi) : \pi \in \Pi) \)

probability dist. over policies \( \pi \) in the policy class \( \Pi \)
Hedging over policies

Get action distributions via policy distributions.

\((Q, x)\)  \(\mapsto\)  \(p\)

(policy distribution, context)  action distribution

1: Pick initial distribution \(Q_1\) over policies \(\Pi\).
2: for round \(t = 1, 2, \ldots\) do
3:   Nature draws \((x_t, r_t)\) from dist. \(D\) over \(X \times [0, 1]^A\).
4:   Observe context \(x_t\).
5:   Compute distribution \(p_t\) over \(A\) (using \(Q_t\) and \(x_t\)).
6:   Pick action \(a_t \sim p_t\).
7:   Collect reward \(r_t(a_t)\).
8:   Compute new distribution \(Q_{t+1}\) over policies \(\Pi\).
9: end for
2. Efficient construction of good policy distributions
Q: How do we choose $Q_t$ for good exploration/exploitation?
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**Caveat:** $Q_t$ must be efficiently computable + representable!
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Our approach:

1. Define convex feasibility problem (over distributions $Q$ on $\Pi$) such that solutions yield (near) optimal regret bounds.
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2. Design algorithm that finds a sparse solution $Q$.

   Algorithm only accesses $\Pi$ via calls to AMO
   $\implies$ $\text{nnz}(Q) = O(# \text{ AMO calls})$
The “good policy distribution” problem

Convex feasibility problem for policy distribution $Q$
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Convex feasibility problem for policy distribution $Q$

$$\sum_{\pi \in \Pi} Q(\pi) \cdot \hat{\text{Reg}}_t(\pi) \leq \sqrt{\frac{K \log N}{t}}$$  \hspace{1cm} \text{(Low regret)}
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Using feasible $Q_t$ in round $t$ gives near-optimal regret.

But $|\Pi|$ variables and $>|\Pi|$ constraints, ...
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Solving the convex feasibility problem

Solver for “good policy distribution” problem

1. If “low regret” constraint violated, then fix by rescaling: $Q := cQ$ for some $c < 1$.
2. Find most violated “low variance” constraint—say, corresponding to policy $\tilde{\pi}$—and update $Q(\tilde{\pi}) := Q(\tilde{\pi}) + \alpha$. (If no such violated constraint, stop and return $Q$.)

($c < 1$ and $\alpha > 0$ have closed-form formulae.)

(Technical detail: $Q$ can be a sub-distribution that sums to less than one.)
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   (Technical detail: $Q$ can be a sub-distribution that sums to less than one.)
Implementation via AMO

Finding “low variance” constraint violation:

1. Create fictitious rewards for each $i = 1, 2, \ldots, t$:

   $$\tilde{r}_i(a) := \hat{r}_i(a) + \frac{\mu}{Q(a|x_i)} \quad \forall a \in \mathcal{A},$$

   where $\mu \approx \sqrt{(\log N)/(Kt)}$.

2. Obtain $\tilde{\pi} := \text{AMO}\left(\{(x_i, \tilde{r}_i)\}_{i=1}^t\right)$.

3. $\text{Rew}_t(\tilde{\pi}) > \text{threshold}$ iff $\tilde{\pi}$’s “low variance” constraint is violated.
Iteration bound

Solver is coordinate descent for minimizing potential function

\[ \Phi(Q) := c_1 \cdot \hat{E}_x[\text{RE}(\text{uniform}||Q(\cdot|x))] + c_2 \cdot \sum_{\pi \in \Pi} Q(\pi)\hat{\text{Reg}}_t(\pi). \]

(Actually use \((1 - \varepsilon) \cdot Q + \varepsilon \cdot \text{uniform}\) inside RE expression.)
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(Partial derivative w.r.t. $Q(\pi)$ is “low variance” constraint for $\pi$.)

Returns a feasible solution after

$$\tilde{O}\left(\sqrt{\frac{Kt}{\log N}}\right) \text{ steps}.$$ 

(Actually use $(1 - \varepsilon) \cdot Q + \varepsilon \cdot \text{uniform}$ inside RE expression.)
Algorithm

1: Pick initial distribution $Q_1$ over policies $\Pi$.
2: for round $t = 1, 2, \ldots$ do
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6: Pick action $a_t \sim p_t$.
7: Collect reward $r_t(a_t)$.
8: Compute new policy distribution $Q_{t+1}$ using coordinate descent + AMO.
9: end for
Recap

Feasible solution to "good policy distribution problem" gives near optimal regret bound.

New coordinate descent algorithm: repeatedly find a violated constraint and adjust $Q$ to satisfy it.

Analysis:
In round $t$, $\text{nnz}(Q_t + 1) = O(\text{AMO calls}) = \tilde{O} \left( \sqrt{Kt \log N} \right)$. 


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New coordinate descent algorithm:
repeatedly find a violated constraint and adjust $Q$ to satisfy it.

Analysis:
In round $t$,

$$\text{nnz}(Q_{t+1}) = O(\# \text{ AMO calls}) = \tilde{O}\left(\sqrt{\frac{Kt}{\log N}}\right).$$
3. Additional tricks: warm-start and epoch structure
Total complexity over all rounds

In round $t$, coordinate descent for computing $Q_{t+1}$ requires $\tilde{O}\left(\sqrt{\frac{Kt}{\log N}}\right)$ AMO calls.
Total complexity over all rounds

In round $t$, coordinate descent for computing $Q_{t+1}$ requires

$$\tilde{O}\left(\sqrt{\frac{Kt}{\log N}}\right)$$

AMO calls.

To compute $Q_{t+1}$ in all rounds $t = 1, 2, \ldots, T$, need

$$\tilde{O}\left(\sqrt{\frac{K}{\log N} T^{1.5}}\right)$$

AMO calls over $T$ rounds.
Warm start

To compute $Q_{t+1}$ using coordinate descent, initialize with $Q_t$. 
Warm start

To compute $Q_{t+1}$ using coordinate descent, initialize with $Q_t$.

1. Total epoch-to-epoch increase in potential is $\tilde{O}(\sqrt{T/K})$ over all $T$ rounds (w.h.p.—exploiting i.i.d. assumption).
Warm start

To compute $Q_{t+1}$ using coordinate descent, initialize with $Q_t$.

1. Total epoch-to-epoch increase in potential is $\tilde{O}(\sqrt{TK})$ over all $T$ rounds (w.h.p.—exploiting i.i.d. assumption).

2. Each coordinate descent step decreases potential by $\Omega\left(\frac{\log N}{K}\right)$.
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To compute $Q_{t+1}$ using coordinate descent, initialize with $Q_t$.

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3. Over all $T$ rounds,

$$\text{total \ # \ calls \ to \ AMO} \leq \tilde{O}\left(\sqrt{\frac{KT}{\log N}}\right)$$
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$$\text{total \# calls to AMO} \leq \tilde{O}\left(\sqrt{\frac{KT}{\log N}}\right)$$

But still need an AMO call to even check if $Q_t$ is feasible!
Epoch trick

Regret analysis: $Q_t$ has low instantaneous per-round regret—this also crucially relies on i.i.d. assumption.
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Doubling: only update on rounds $2^1, 2^2, 2^3, 2^4, \ldots$

$log T$ updates, so $\tilde{O}(\sqrt{KT/\log N})$ AMO calls overall.
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Regret analysis: $Q_t$ has low instantaneous per-round regret—this also crucially relies on i.i.d. assumption.

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Doubling: only update on rounds $2^1$, $2^2$, $2^3$, $2^4$, ... 
$log T$ updates, so $\tilde{O}(\sqrt{KT}/\log N)$ AMO calls overall.

Squares: only update on rounds $1^2$, $2^2$, $3^2$, $4^2$, ... 
$\sqrt{T}$ updates, so $\tilde{O}(\sqrt{K}/\log N)$ AMO calls per update, on average.
Warm start + epoch trick

Over all $T$ rounds:

- Update policy distribution on rounds $1^2$, $2^2$, $3^2$, $4^2$, $\ldots$, i.e., total of $\sqrt{T}$ times.

- Total # calls to AMO:

\[ \tilde{O}\left(\sqrt{\frac{KT}{\log N}}\right). \]

- # AMO calls per update (on average):

\[ \tilde{O}\left(\sqrt{\frac{K}{\log N}}\right). \]
4. Closing remarks and open problems
Recap

1. New algorithm for general contextual bandits
2. Accesses policy class $\Pi$ only via AMO.
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Open problems

1. Empirical evaluation.
3. Alternatives to AMO.
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Thanks!
Given policy distribution \( Q \) and context \( x \),

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\forall a \in A. \quad Q(a|x) := \sum_{\pi \in \Pi} Q(\pi) \cdot 1\{\pi(x) = a\}
\]

(so \( Q \mapsto Q(\cdot|x) \) is a linear map).
Projections of policy distributions

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$$\forall a \in A. \quad Q(a|x) := \sum_{\pi \in \Pi} Q(\pi) \cdot 1\{\pi(x) = a\}$$

(so $Q \mapsto Q(\cdot|x)$ is a linear map).

We actually use

$$p_t := Q_t^{\mu_t}(\cdot|x_t) := (1 - K\mu_t)Q_t(\cdot|x_t) + \mu_t 1$$

so every action has probability at least $\mu_t$ (to be determined).
The potential function

\[ \Phi(Q) := t_\mu_t \left( \frac{\hat{E}_{x \in H_t} \left[ \text{RE}(\text{uniform} \| Q^{\mu_t}(\cdot|x)) \right]}{1 - K_\mu_t} + \sum_{\pi \in \Pi} \frac{Q(\pi)\hat{\text{Reg}}_t(\pi)}{K_t \cdot \mu_t} \right), \]