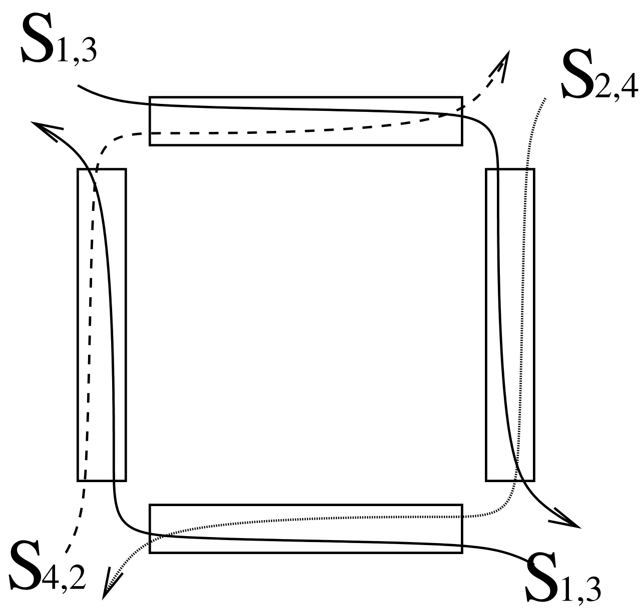
# HW #8

ELEN E4710 - Intro to Network Engineering — Due 12/8/2003
Fall 2003 — Prof. Rubenstein

Homework must be turned in at the beginning of class on the due date indicated above. CVN students have one additional day. Late assignments will not be accepted.




1. Consider the networking system depicted above where each link has capacity $C$ and each flow traverses two hops, with each source transmitting data at the same rate, $\rho > C/2$. Assume that each router is bufferless (i.e., if a packet comes when another packet is being processed in the router's buffer, it must be dropped), and that the router uses virtual clock to schedule packets. Using a fluid model, where $\rho$ is the rate at which each flow transmits into the network, let $\gamma$ be the rate at which a single flow exits the first hop of the transmission, and let $\lambda$ be the rate at which flow exits the second hop. Let $A_\rho$ be the fraction of first-hop flow that is accepted (i.e., each flow is rejected from the first hop of the network at rate $(1-A_\rho)\rho$) and let $A_\gamma$ be the fraction of second-hop flow that is accepted.

   (a) Give the equation for $C$ as a function of $\rho, \gamma, A_\rho, A_\gamma$.

   (b) Give equations for

      i. $\gamma$ as a function of $\rho$ and $A_\rho$.

      ii. $\lambda$ as a function of $\gamma$ and $A_\gamma$.

      Not all variables listed need to appear in the equation. The equations in this part should not depend on $C$.

   (c) Explain why we cannot have $A_\gamma\gamma > A_\rho\rho$. (Hint: this holds true even without virtual clock, so you do not need to use the fact that the routing uses virtual clock here).

   (d) Prove that we cannot have $A_\gamma\gamma < A_\rho\rho$?

   (e) Since we must have$A_\gamma\gamma = A_\rho\rho$, use the formulas computed in parts 1a and 1b to compute $\lambda$.

   (f) Using the results above, indicate whether congestion collapse can occur in the above system when flows transmit at a constant rate $\rho$ and virtual clock is the queueing discipline applied.

2. Consider a flow that sends packets at a fixed rate, $\alpha$ through a router that applies a leaky bucket congestion control mechanism, where tokens enter the leaky bucket at rate $\rho$, and the bucket has size $b$. The flow bundles together $k$ packets and sends these packets to the router every $s = \alpha/k$ seconds.

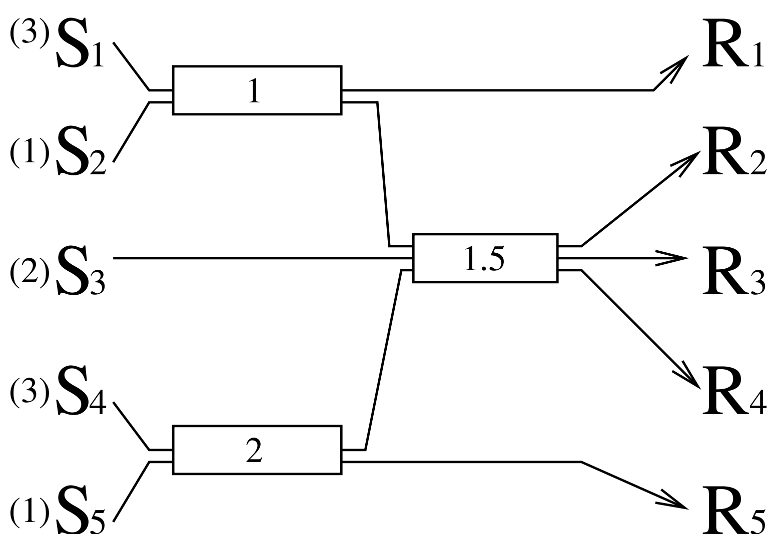
   (a) In terms of $s, \rho, b$, and $k$, what is the long-term rate of the flow? More precisely, let $n(t)$ be the number of packets that have passed through the router by time $t$, where $n(0) = 0$ and the flow starts transmitting at time $t = 0$. What is $\lim_{t\to\infty} n(t)/t$?

   (b) Prove that setting $k = 1$ (and hence $s = \alpha$) achieves the highest possible rate.

   (c) It is likely that this highest possible rate can also be achieved for other values of $k$. What is the maximum value of $k$, where $s = \alpha/k$ that achieves the same rate through the router?

3. In a **weighted** max-min fair allocation, each flow $f_i$ is assigned a weight $w_i$ such that an allocation of rates $(\rho_1, \cdots, \rho_n)$ to flows $f_1, \cdots, f_n$ is weighted max-min fair whenever the allocation is feasible and if, for any $\epsilon$, any flow $f_i$'s rate is increased to $\rho_i + \epsilon$, to produce a feasible allocation, another flow $f_j$'s rate must be decreased where $\rho_j/w_j \le \rho_i/w_i$. Note that when all weights are set to identical values, the weighted max-min fair allocation equals the max-min fair allocation.

   (a) Consider three flows $f_1, f_2, f_3$ that are assigned weights 1,3,6 respectively. If the network consists of a single link with capacity 10 and all three flows traverse this link, what are the respective rates within the weighted max-min fair allocation?




   (b) In the figure above, if all flow weights are 1, what is the max-min fair allocation?

   (c) In the figure above, the flow weights are indicated in parentheses next to the source of the flow. What is the weighted max-min fair allocation in this network?

   (d) Give a one-sentence description of how the algorithm that computes the max-min fair allocation needs to be modified to produce the weighted max-min fair allocation.