1. Construct a FSM for a router that implements weighted fair queueing for two flows $f_1$ and $f_2$ where $w_1 = 4$, $w_2 = 3$, and the slack is $1/3$ for both flows (where the processing of a packet from $f_1$ adds $1/4$ to its virtual clock, and the processing of a packet from $f_2$ adds $1/3$ to its virtual clock). Your state machine can depend on the following functions and events to simplify its design:

- $IE()$: the queue is empty (prior to an arrival)
- $H(i) : H(i)$ equals 0 when no packets from $f_i$ are queued in the system, and equals 1 otherwise
- $D()$: triggered when the router completes processing its current packet.
- $A(i)$: triggered when an arrival to the router from $f_i$ occurs (only needs to be used when the router is not processing any packets).

With these functions and events, you should build your FSM so that it indicates clearly whose packet should be processed next whenever such a decision needs to be made. (Hint: each state should indicate the current difference between the two flows’ clocks)

2. Consider a queue where packets arrive at rate $\lambda$ and whose processing times are exponentially distributed with rate $\mu$. When the queue is empty, the processor processes “pretend” packets at rate $\mu$. When a real packet arrives, the pretend packet is immediately aborted (discarded) and the real packet is processed. The completion of a “pretend” packet is counted as an event.

Recall that $S(n)$ equals the number of packets in the system after the $n$th event. Here, an event is a packet arrival or a service completion, including the completion of pretend packets.

(a) What are $Pr(S(n) = 1|S(n-1) = 0)$ and $Pr(S(n) = 0|S(n-1) = 0)$?

(b) Construct the Markov model for this queueing system.

(c) Solve for the steady-state distribution, $\pi_i = \lim_{n \to \infty} Pr(S(n) = i)$ for all $i$, $0 \leq i \leq k$.

(d) Solve for $\pi_i$ as $k \to \infty$.

(e) $\pi_i$ is different than the value computed using the Markov model formulated in class. Explain why this is the case.

3. Construct a Markov model for a round-robin queueing system that can store up to 3 packets for processing. Assume there are two flows in the system, where both flow’s packets are processed at rate $\mu$, and flow $f_i$’s packets arrive at rate $\lambda_i$. Label transitions with their transition probabilities.
4. Construct Markov models for each of two priority queueing systems, each processing packets from two flows. Each system can hold a total of $k$ packets from each flow. A packet from $f_1$ should always be processed before any packets from $f_2$ in the queue. Assume both flow’s packets are processed at rate $\mu$, and flow $f_i$’s packets arrive at rate $\lambda_i$.

(a) In the first system, a packet from $f_1$ arriving to a full queue will replace a packet in the queue from $f_2$ (if one exists).

(b) In the second system, packets in the queue are not removed except when their processing is complete (i.e., a packet arriving from $f_1$ to a full queue will be turned away, even if there are packets from flow $f_2$ in the queue.)