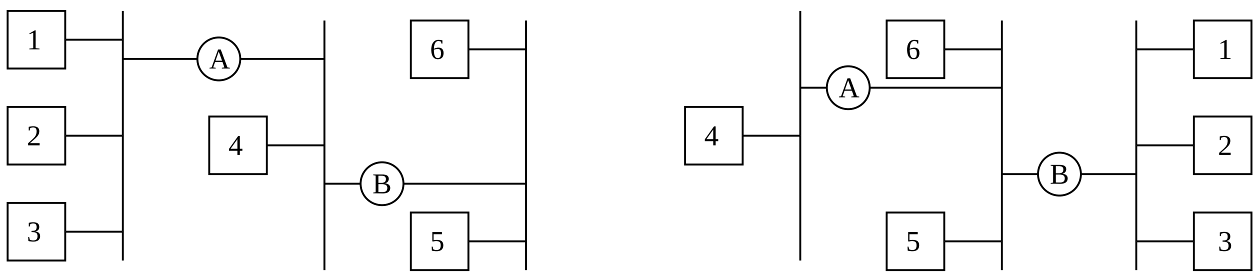
HW #3

ELEN E4710 - Intro to Network Engineering | Due 2/27/2003
Spring 2002 | Prof. Rubenstein

Homework must be turned in at the beginning of class on the due date indicated above. CVN students have one additional day. Late assignments will not be accepted.

1. Trains arrive at the 116th Street station at rate λ. Whenever a train arrives, the number of people that get off the train is uniformly distributed between 10 and 100. Only one exit gate is opened (so that all people exiting must pass through the same gate), and the line at the gate has 12 people waiting on average. What is the expected time a random person needs to wait to exit the station?



(a) Configuration 1 (b) Configuration 2

2. Consider the above two configurations of a LAN, where each square is a transmitting device. Suppose transmissions are slotted, and let $T_{i,j}$ be a random variable that equals 1 if device i transmits during slot j and 0 otherwise. Let $S_{i,j}$ be 1 if $T_{i,j} = 1$ and the transmission was successful, and equal 0 otherwise. Let $P(T_{i,j} = 1) = p$ for all i, j.

   (a) Suppose A and B are hubs. What is $P(S_{1,j} = 1|T_{1,j} = 1)$ in each of the configurations?

   (b) Suppose A is a hub and B is a switch. What is $P(S_{1,j} = 1|T_{1,j} = 1)$ in each configuration?

   (c) Suppose A and B are both switches. What is $P(S_{1,j} = 1|T_{1,j} = 1)$ in each configuration?

3. $2N$ devices numbered $1, \cdots, 2N$ share a LAN and frame transmission times are segmented into slots such that two transmissions during the same slot always cause transmission failures, whereas two transmissions during different slots do not cause transmission failures. Assume that each device has a frame to send every time-slot, but uses the following rule to decide whether or not to try to send:

   - If the device is odd-numbered, it transmits with probability p_1.
   - If the device is even-numbered, it can sense the line and see whether an odd-numbered device is transmitting. If not, the device transmits with probability p_2.

   (a) What is the probability of a successful transmission in a given timetick by device 1 (in terms of N, p_1, p_2)?

   (b) What is the probability of a successful transmission in a given timetick by device 2 (in terms of N, p_1, p_2)?

   (c) What is the probability of a successful transmission by any device?

   (d) What is the expected number of successful transmissions per time-slot?

   (e) What value of p_1 (in terms of N) maximizes the rate at which device 1 can successfully transmits (i.e., maximizes the probability that it successfully transmits within a given slot)?

(f) Given p_1 is set to the value chosen above, what value of p_2 (in terms of N) maximizes the rate at which device 2 can successfully transmits (i.e., maximizes the probability that it successfully transmits within a given slot)?

(g) In your solution above, odd devices should have an advantage over even devices, in that odd devices probabilities of a succesful transmission should be higher than their even counterparts. Is this true, no matter how p_1 and p_2 are set? Explain why or why not.

4. Suppose that n devices share a LAN, where each device sends frames that take L microseconds to transmit onto the wire, with $L > 2\tau$ where τ is the maximum propagation delay on the LAN. k of these n devices use ALOHA, where the backoff occurs at rate λ, the other $n - k$ devices use slotted ALOHA with slots of size L and backoff at rate λ. What is the probability of successful transmission for

   (a) A device using ALOHA

   (b) A device using slotted ALOHA

   (c) A device drawn at uniformly at random from the set of devices.

5. Device 1 and Device 2 share a common LAN on which they transmit. They can sense each other's signals in time $\tau = 0$. Device i's transmissions last a length of time that are exponentially distributed with length μ_i. The devices use CSMA to ensure that collisions never occur by sensing the line right before they start a transmission, and backing off for a time that is exponentially distributed with rate λ_i if the line is busy before checking the line again. When a device successfully completes a transmission, it follows its transmission with an exponential backoff at rate λ_i before initiating its next transmission.

   What value should λ_2 take (as a function of λ_1, μ_1, and μ_2 to ensure that device 2 has the same likelihood of being the next device to transmit from some arbitrary point in time, t?

6. Let C_1, C_2, C_3 be three connections that use CDMA to transmit upon the same channel using chipping signals (1,1,1,1,1,1,1,1), (1,1,-1,-1,1,1,-1,-1), and (1,1,-1,-1,-1,-1,1,1) respectively. If the received chipping signal is (-3,-3,1,1,-1,-1,-1,-1), what was the value of the bit transmitted by connections C_1, C_2, C_3 (where the value is either 1 or -1)?