HW #2

ELEN E4710 - Intro to Network Engineering — Due 2/13/2003
Spring 2002 — Prof. Rubenstein

Homework must be turned in at the beginning of class on the due date indicated above. CVN students have one additional day. Late assignments will not be accepted.

1. You work in a department store that has a Santa Claus who sits children on his lap and asks them what they want for Christmas. Having parents and their children simply waiting for Santa means that they will spend less time shopping! Suppose customers arrive to shop at a rate of 10 shoppers per minute. To keep the line to a length of less 50 shoppers, what is the maximum average time that Santa should spend with each customer?

2. Let $X_1, X_2, \cdots, X_k$ be independent, exponentially distributed random variables with rate λ and let $Y = \max\{X_1, X_2, \cdots, X_k\}$. What is $E[Y]$?

3. You arrive at a bus stop at 3:15 pm. Two busses, one red, one blue are both scheduled to arrive independently according to a uniform distribution between 3 and 4 pm.
   (a) What is the probability that you missed both buses?
   (b) Somebody informs you that the red bus has not yet arrived, but is unsure about the blue bus. Given this information, what is the probability that either comes within the next 15 minutes?

4. You arrive at a bus stop at 3:15 pm. Two busses, one red, one blue, arrive according to an exponential distribution with rate λ, starting at 3pm (they will not arrive before then).
   (a) What is the probability that you missed both buses?
   (b) Somebody informs you that the red bus has not yet arrived. Given this information, what is the probability that either bus comes within the next 15 minutes?
   (c) Suppose the red bus arrives at rate μ and the blue bus arrives at rate λ. Given the same information as in part (4b), what is the probability that the next bus to arrive is red?

5. Suppose that the arrival times of three buses numbered 1,2,3 are exponentially distributed with rates λ_1, λ_2, and λ_3 (here, the any bus can come first, i.e., the clocks for all buses start at time 0).
   (a) What is the probability that buses arrive in the order 1,2,3?
   (b) What is the probability that no bus arrives within time t?
   (c) What is the probability that exactly 2 buses arrive before time t?
   (d) What is the probability that exactly 1 bus arrives before time t, given that no buses arrive before time $s < t$?
   (e) What is the probability that exactly 2 buses arrive before time t, given that all buses arrive before time $s > t$?

6. Consider a 2-D parity check code.

   (a) Prove that any combination of 1, 2, or 3 bit errors is detectable.

   (b) For a 16-bit data word (i.e., code bits are extra bits), show a 6 bit error combination that cannot be detected, and show one that can.

   (c) For a data word with k^2 bits, given that bit flips are Bernoulli (independent) with probability p, compute the probability that 6 bit-errors occur and that the 2-D parity check fails to detect that the word is corrupted.

$$\begin{array}{cccc|c}
1 & 0 & 1 & 1 & 1 \\
0 & 1 & 0 & 0 & 1 \\
0 & 1 & 1 & 1 & 0 \\
1 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 1 & 0 & 0
\end{array}$$

7. Fix the bit error in the above codeword.

8. Suppose you receive the codeword 1001100 which was generated using the (7,4) Linear code whose code bits are generated as follows:

   - $c_1 = b_1 \oplus b_3 \oplus b_4$
   - $c_2 = b_1 \oplus b_2 \oplus b_4$
   - $c_3 = b_2 \oplus b_3 \oplus b_4$

   (a) What codeword was most likely transmitted?

   (b) Suppose you are told that the error bits are correct, but that 2 data bits are flipped. Which two are most likely flipped?

   (c) Suppose the error bits are correct. Can 3 data bits be flipped? What about 4? Explain how you know this.

9. Suppose we switch to the linear code:

   - $c_1 = b_1 \oplus b_3$
   - $c_2 = b_2 \oplus b_4$
   - $c_3 = b_2 \oplus b_3$

   (a) Can this code always detect single-bit errors? Explain why or why not.

   (b) List the sets of single-bit errors that should be detected, but not repaired (because 2 possible repairs are equally likely).