HW #1

ELEN E4710 - Intro to Network Engineering | Due 2/4/2002
Spring 2003 | Prof. Rubenstein

Homework must be turned in at the beginning of class on the due date indicated above. CVN students have one additional day. Late assignments will not be accepted.

1. Two fair n-sided die (each side is equally likely to come up) with sides numbered from 1 to n is rolled until the value 10 appears on top. How many rolls are expected?

2. A fair coin is tossed 10 times. What is the probability that either the first five tosses are heads or the first two tosses are heads and the last 5 tosses are tails?

3. A game is of Biased HighCard is played as follows: you select a card and put it back in the deck. Your opponent then selects a card and puts it back. If your card was smaller, you can then select another card. Your opponent cannot select another card, hence the game is biased toward you.

   (a) What is the probability that you will win (the card you wind up with being larger than the opponent's card).

   (b) If you gain a dollar for each win and lose a dollar for each loss, how much money will you have left on average after playing 100 games?

4. Consider a drawer containing n red balls and m blue balls.

   (a) You pick a ball uniformly at random, put it back, and pick again, choosing a ball a total of k times. X is a r.v. that equals the number of times you picked a red ball. What is $P(X = 0)$?

   (b) Suppose you don't put the ball back after each selection. Let Y equal the number of red balls selected. What is $P(Y = 0)$.

   (c) For the case where the ball is returned after each selection, let Z be the number of times that the ith pick and the $i - 1$st pick are the same color. What is $E[Z]$?

   (d) For the case where the ball is not returned after each selection, let W be the number of times that the ith pick and the $i - 1$st pick are the same color. What is $E[W]$? (Hint: sum of expectations equaling expectation of sums is probably very useful here).

5. Let X, Y, and Z be random variables such that X and Y are independent, X and Z are independent, and Y and Z are independent (i.e., $P(X = n, Y = m) = P(X = n)P(Y = m), P(X = n, Z = m) = P(X = n)P(Z = m), P(Y = n, Z = m) = P(Y = n)P(Z = m)$.

   (a) Show that this does not imply that X, Y and Z are independent (i.e., that it is not always true that $P(X = \ell, Y = m, Z = n) = P(X = \ell)P(Y = m)P(Z = n)$). Think about the sum of 2 dice modulo 6...

   (b) Assume Z is an integer-valued random variable. Prove that if X, Y, and Z are independent, then X and Y are independent. (Note: for the proof, start with $P(X = \ell, Y = n, Z = m) = P(X = \ell)P(Y = n)P(Z = m)$ and after some mathematical manipulation, wind up with $P(X = \ell, Y = n) = P(X = \ell)P(Y = n)$).

6. With probability p_1, TA Bhattacharjee has a message to give to students Hans and Frans in the class. With probability p_2, Professor Rubenstein will remember to deliver the message to Hans, and with probability p_3 he gives the message to Frans. If the delivery to Hans and Frans are independent events such that the probability that Hans gets the message is $p_1 p_2$ and that Frans gets the message is $p_1 p_3$, what is the probability that Frans did not get message, given that Hans did not receive one?