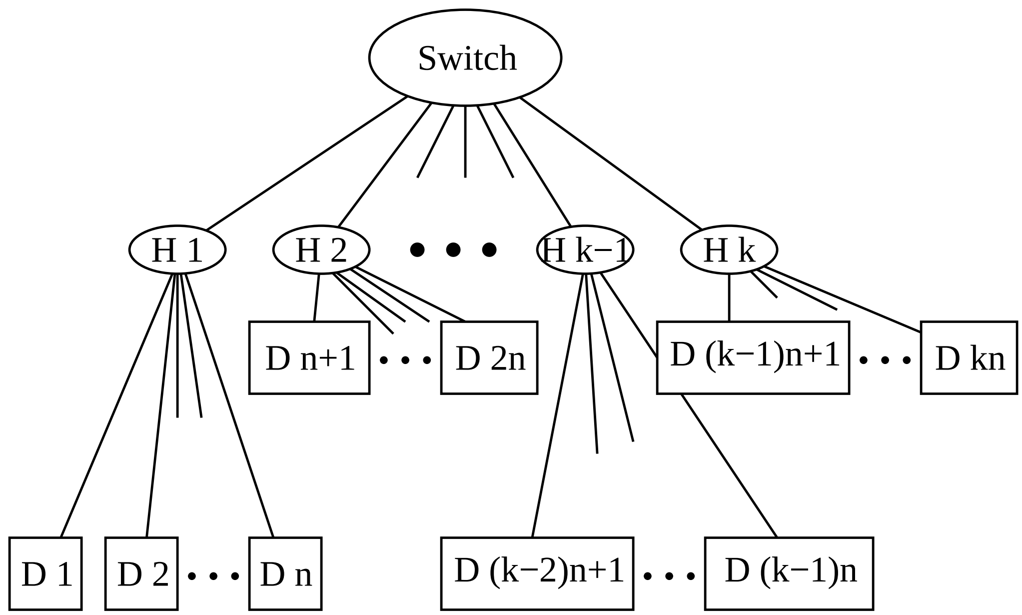
HW #3

ELEN E4710 - Intro to Network Engineering — Due 2/26/2002
Spring 2002 — Prof. Rubenstein

Homework must be turned in at the beginning of class on the due date indicated above. CVN students have one additional day. Late assignments will not be accepted.

1. Trains arrive at the 116th Street station according to an exponential distribution with rate λ. Whenever a train arrives, the number of people that get off the train is uniformly distributed between 10 and 100. Only one exit gate is opened (so that all people exiting must pass through the same gate), and the line at the gate has 12 people waiting on average. What is the expected time a random person needs to wait to exit the station?



2. In class, we started the computation for the configuration shown above of the probability that device 1 makes a successful transmission, given that device 1 transmits, where

   - each device transmits during each clock tick with probability p (where the decision to transmit is a Bernoulli process), and the recipient device of the transmission is chosen uniformly from the set of remaining devices (other than the transmitter).
   - two frames collide on a link ℓ if they are transmitted during the same clock tick and both traverse ℓ
   - a transmission from device i to device j fails if the transmission of the frame from i to j collides with another frame along any link on the path from i to j (collisions that happen off this path do not affect the transmission from i to j).
   - the switch at the top "knows" which hub to transmit to in order for a particular device to be reached. Therefore, when delivering a frame to device j, it forwards the frame only on the link that leads to device j.
   - the other hubs do not have these suppression capabilities and must forward all transmissions on all outgoing links.

   Recall that we looked at a particular clock tick t, and we let T_i be a random variable that equals 1 when device i transmits at time t and 0 otherwise. We let d_i be a random variable that equals the destination of any transmission from device i, and we defined X_i to be a random variable that equals 1 if device i's transmission at time t is successful and 0 otherwise. We therefore wish to compute $\Pr(X_i = 1 | T_i = 1)$. We did this as follows:

$$\begin{aligned}\Pr(X_1=1|T_1=1) &= \Pr(X_1=1,T_1=1)/\Pr(T_1=1)\\ &= (\sum_{j=2}^{n}\Pr(X_1=1,T_1=1,d_1=j)+\sum_{j=n+1}^{kn}\Pr(X_1=1,T_1=1,d_1=j))/p\\ &= \frac{(n-1)p(1-p)^{n-1}\frac{1}{kn-1}((1-p)+p\frac{(k-1)n-1}{kn-1})^{(k-1)n}+\sum_{j=n+1}^{kn}\Pr(X_i=1,T_i=1,d_1=j)}{p}\end{aligned}$$

To complete the computation, you need to compute $\sum_{j=n+1}^{kn}\Pr(X_1=1,T_1=1,d_1=j)$.

3. Suppose a switch is used in place of hub 1 (the hub immediately above device 1) and the switch at the top is replaced by a hub.

   (a) Give the set of transmission scenarios where device 1's transmission would fail under the original configuration but succeeds here.

   (b) Give the set of transmission scenarios where devices 1's transmission would succeed under the original configuration but would fail here.

   (c) Compute $\Pr(X_1=1|T_1=1)$

4. N devices share a LAN and frame transmission times are segmented into slots such that two transmissions during the same slot always cause transmission failures, whereas two transmissions during different slots do not cause transmission failures. Assume that each device has a frame to send every time-slot, but only performs the transmission with a probability p.

   (a) What is the probability of a successful transmission in a given timetick (in terms of N and p).

   (b) What is the probability that device 1's transmission is successful, given device 1 attempts a transmission?

   (c) What is the expected number of successful transmissions per time-slot?

   (d) What value of p (in terms of N) maximizes the probability in part 4a.

5. Suppose that n devices share a LAN, where each device sends frames that take L microseconds to transmit onto the wire, with $L > 2\tau$ where τ is the maximum propagation delay on the LAN. k of these n devices use ALOHA, where the backoff occurs at rate λ, the other $n-k$ devices use slotted ALOHA with slots of size L and backoff at rate λ. What is the probability of successful transmission for

   (a) A device using ALOHA

   (b) A device using slotted ALOHA

   (c) A device drawn at uniformly at random from the set of devices.

6. A set of devices on a LAN transmit frames whose lengths are exponentially distributed with rate λ_1. The system used to use non-persistent CSMA where a device would check if the wire was available at the instant it wanted to transmit and send the frame if the wire was free. If not, the device would back off for a time t that was exponentially distributed with rate λ_2. When time t completed, the process would repeat until the frame was sent.

   The system is being switched to one where the line can be sensed continuously, such that the precise end of the previous transmission can be determined. A backoff whose time is exponentially distributed with rate λ_3 is applied once the wire goes free to prevent synchronized frame transmissions. Compute λ_3 as a function of λ_2 and λ_1 such that the expected transmission rates (i.e., the distribution on how long the backoff is from the end of a transmission frame) are the same in the two systems. Show or explain how you get your result.

7. Let C_1, C_2, C_3 be three connections that use CDMA to transmit upon the same channel using chipping signals (1,1,1,1,1,1,1,1), (1,1,-1,-1,1,1,-1,-1), and (1,1,-1,-1,-1,-1,1,1) respectively. If the received chipping signal is (1,1,1,1,-1,-1,3,3), what was the value of the bit transmitted by connections C_1, C_2, C_3 (where the value is either 1 or -1)?