HW #2

ELEN E4710 - Intro to Network Engineering — Due 2/12/2002
Spring 2002 — Prof. Rubenstein

Homework must be turned in at the beginning of class on the due date indicated above. CVN students have one additional day. Late assignments will not be accepted.

1. You work in a department store that has a Santa Claus who sits children on his lap and asks them what they want for Christmas. Parents complain that on average, they wait 50 minutes to see Santa and on average spend only 30 seconds with Santa. You are unable to determine the distribution on the time that Santa spends with children. Can you still calculate the expected length of the line? If so, how and what is it? In not, why not?

2. Let $X_1, X_2, \cdots, X_k$ be independent, exponentially distributed random variables with rate λ.
   Let $Y = \min\{X_1, \cdots, X_k\}$. Show that Y is exponentially distributed. What is the rate of Y?

3. You arrive at a bus stop at 3:15 pm. A bus arrives according to a uniform distribution between 3 and 4 pm.

   (a) What is the probability that you missed the bus?

   (b) Somebody informs you that the bus has not yet arrived. Given this information, what is the probability that it comes within the next 15 minutes?

4. You arrive at a bus stop at 3:15 pm. A bus arrives according to an exponential distribution with rate λ, starting at 3pm (it will not arrive before then).

   (a) What is the probability that you missed the bus?

   (b) Somebody informs you that the bus has not yet arrived. Given this information, what is the probability that it comes within the next 15 minutes?

   (c) Suppose there are 3 buses numbered 1,2,3 where the first bus arrives at a time after 3pm that is exponentially distributed with rate λ. The second bus arrives at a time after the arrival of the first bus that is exponentially distributed with rate λ, and the third bus arrives at a time after the second bus that is exponentially distributed with rate λ. What is the probability that the third bus takes more than an hour to arrive?

5. Suppose that the arrival times of three buses numbered 1,2,3 are exponentially distributed with rates λ_1, λ_2, and λ_3 (here, the any bus can come first, i.e., the clocks for all buses start at time 0).

   (a) For the case where $\lambda_1 = \lambda_2 = \lambda_3 = \lambda$, What is the probability that it takes more than an hour for all three buses to arrive?

   (b) What is the probability that buses arrive in the order 1,2,3?

   (c) What is the probability that bus 2 arrives more than time t after bus 1, given that bus 2 arrives after bus 1?

   (d) What is the probability that bus 2 takes more than time t to arrive given that bus 1 comes after bus 2?

6. Consider a 2-D parity check code.

   (a) Prove that any combination of 1, 2, or 3 bit errors is detectable.

   (b) For a 16-bit data word (i.e., code bits are extra bits), show a 4 bit error combination that cannot be detected, and show one that can.

   (c) For a data word with k^2 bits, given that bit flips are a Bernoulli process (independent) with probability p, compute the probability that 4 bit-errors occur and that the 2-D parity check fails to detect that the word is corrupted.

$$\begin{array}{cccc|c} 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ \hline 1 & 0 & 0 & 0 & 1 \end{array}$$

   (d) Fix the bit error in the above codeword.

7. Consider the (7,4) Linear code whose code bits are generated as follows:

   - $c_1 = b_1 \oplus b_3 \oplus b_4$
   - $c_2 = b_1 \oplus b_2 \oplus b_4$
   - $c_3 = b_2 \oplus b_3 \oplus b_4$

   Suppose you receive the codeword 1110111 which was generated using the above linear code. What codeword was most likely transmitted?

8. The above codes work best when p, the probability of a bit being flipped, is very small. What if p were very large (e.g., $1 - \epsilon$ for some very small ϵ) and Bernoulli. How would you modify the coding technique to get guarantees that were as good as if p were very small (i.e., $p = \epsilon$)?

9. Suppose we switch to the linear code:

   - $c_1 = b_1 \oplus b_3$
   - $c_2 = b_2 \oplus b_4$
   - $c_3 = b_2 \oplus b_3$

   (a) Can this code always detect single-bit errors? Explain why or why not.

   (b) List the sets of single-bit errors that should be detected, but not repaired (because 2 possible repairs are equally likely).