HW #1

ELEN E4710 - Intro to Network Engineering — Due 1/31/2002
Spring 2002 — Prof. Rubenstein

Homework must be turned in at the beginning of class on the due date indicated above. CVN students have one additional day. Late assignments will not be accepted.

1. A fair n-sided die (each side is equally likely to come up) with sides numbered from 1 to n is rolled until the value 5 appears on top. How many rolls are expected?

2. A fair coin is tossed 10 times. What is the probability that either the first five tosses are heads or the last 5 tosses are tails?

3. You lose at craps if on your first roll, you roll 2,3 or 12. You win if on your first roll, you roll 7 or 11. You win if you roll anything else (4-6, 8-10) on your first roll and roll this value again before rolling a 7. Otherwise you lose.

   (a) What is the probability that you will win? If you do this right, the odds should be in the house's favor (i.e., less than .5), but by very little.

   (b) If you gain a dollar for each win and lose a dollar for each loss, how much money will you have left on average after playing 100 games?

4. Consider a room containing n people in which each person is equally likely to be born on any day of the year (i.e., assume no leap years and $P(G_i = k) = 1/365$, where G_i is the birth date of the ith person and k is a number between 1 and 365 indicating that person's birth date).

   (a) What is the probability that no two people have the same birthday?

   (b) What is the probability that k people were born on January 24 where $k < n$?

   (c) How many pairs of people can we expect to have the same birthday? Give a closed-form solution without summations.

5. Let X, Y, and Z be random variables such that X and Y are independent, X and Z are independent, and Y and Z are independent (i.e., $P(X,Y) = P(X)P(Y), P(X,Z) = P(X)P(Z), P(Y,Z) = P(Y)P(Z)$. Show that this does not imply that X, Y and Z are independent (i.e., that it is not always true that $P(X,Y,Z) = P(X)P(Y)P(Z)$). Think about rolling 2 dice...

6. With probability p_1, TA Stavrou has a message to give to the class. With probability p_2, Professor Rubenstein will remember to deliver the message. With probability p_3, Mary, a student in the class arrives on time to hear any announcements Professor Rubenstein makes. Assuming these three events are independent (such that the probability that Mary hears an announcement is $p_1p_2p_3$), what is the probability that TA Stavrou gave an announcement to Professor Rubenstein, given that Mary did not hear an announcement?