

1) $N = \lambda T$ is shorthand for $E[N] = E[\lambda] E[T]$

if trains arrive at rate λ , and each arrival brings on average $\frac{100+10}{2} = 55$ people, then

$$E[\lambda] = \lambda \cdot 55$$

$$E[N] = 12 \quad \therefore E[T] = \frac{12}{\lambda \cdot 55}$$

2) a) $P(\text{successful xmission} | 1 \text{ xmits}) = P(\text{no other device xmits}) = (1-p)^5$ for both configurations

b) Configuration 1: $P(\text{success} | 1 \text{ xmits}) = P(1 \text{ xmits to } 2,3,4 \text{ AND } (2,3,4 \text{ don't xmit AND } (5,6 \text{ don't xmit OR } 5,6 \text{ just xmit to each other}))$
 OR 1 xmits to 5,6 AND (no one else xmits))
 $= \frac{3}{5} \cdot (1-p)^3 [(1-p) + p \cdot \frac{1}{5}]^2 + \frac{2}{5} (1-p)^5$

Config 2: $P(\text{success} | 1 \text{ xmits}) = P(1 \text{ xmits to } 2,3 \text{ AND } (2,3 \text{ don't xmit AND } (4,5,6 \text{ don't xmit OR } 4,5,6 \text{ just xmit to each other}))$
 OR 1 xmits to 4,5,6 AND (no one else xmits))
 $= \frac{2}{5} (1-p)^2 [(1-p) + p \cdot \frac{2}{5}]^3 + \frac{3}{5} (1-p)^5$

c) Config 1: $P(\text{success} | 1 \text{ xmits}) = P(1 \text{ xmits to } 2,3 \text{ AND } (2,3 \text{ don't xmit AND } (4,5,6 \text{ don't xmit OR } 4,5,6 \text{ just xmit to each other}))$
 OR 1 xmits to 4 AND (2,3,4 don't xmit AND (5,6 don't xmit OR 5,6 just xmit to each other))
 OR 1 xmits to 5,6 AND no one else xmits)
 $= \frac{2}{5} (1-p)^2 [(1-p) + p \cdot \frac{2}{5}]^3 + \frac{1}{5} (1-p)^3 [(1-p) + p \cdot \frac{1}{5}]^2 + \frac{2}{5} (1-p)^5$

Config 2: $P(\text{success} | 1 \text{ xmits}) = P(1 \text{ xmits to } 2,3 \text{ AND } (2,3 \text{ don't xmit AND } (4,5,6 \text{ don't xmit OR } 4,5,6 \text{ just xmit to each other}))$
 OR 1 xmits to 4,5,6 AND (no one else xmits))
 $= \frac{2}{5} (1-p)^2 [(1-p) + p \cdot \frac{2}{5}]^3 + \frac{3}{5} (1-p)^5$

$$3) a) P(\text{successful xmission by device 1}) = p_1 \cdot P(\text{no other odd device xmits AND no even device xmits}) \\ = p_1 (1-p_1)^{N-1} \cdot 1^N = p_1 (1-p_1)^{N-1}$$

$$b) \quad " \quad " \quad " \quad 2) = p_2 \cdot P(\text{no odd device xmits AND no other even device xmits}) \\ = p_2 (1-p_1)^N \cdot (1-p_2)^{N-1}$$

$$c) P(\text{successful xmission}) = P(2 \text{ is successful}) + P(4 \text{ is successful}) + \dots + P(1 \text{ is successful}) + P(3 \text{ is successful}) + \dots \\ = N p_1 (1-p_1)^N (1-p_2)^{N-1} + N p_2 (1-p_1)^{N-1} (1-p_2)^N$$

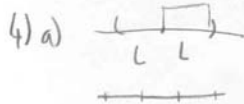
$$d) E[X_{\text{missions}}] = 1 \cdot P(1 \text{ successful xmission}) + 0 \cdot P(\text{no successful xmission}) = P(\text{successful xmission}) \\ (\text{as defined in part c})$$

$$e) p(1-p)^{x-1} \text{ maximized when } p = \frac{1}{x} \quad \boxed{\text{PROOF: } \frac{d}{dp} p(1-p)^{x-1} = 0 \quad -p(x-1)(1-p)^{x-2} + (1-p)^{x-1} = 0 \quad p=1 \text{ or } \dots \\ p(1-x) + 1-p = 0 \quad 1-x-1 = -\frac{1}{p} \quad p = \frac{1}{x} \\ (\text{assume } x > 1) \quad 1(1-1)^{x-1} = 0 < \frac{1}{x} (1-\frac{1}{x})^{x-1} \therefore p = \frac{1}{x} \text{ is max}}$$

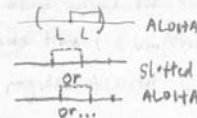
$$p_1 (1-p_1)^{N-1} \text{ maximized when } p_1 = \frac{1}{N} \quad (N > 1)$$

$$f) k p_2 (1-p_2)^{N-1} \text{ is maximized when } p_2 (1-p_2)^{N-1} \text{ maximized (where } k \text{ is positive constant } = (1-p_1)^N) \\ p_2 (1-p_2)^{N-1} \text{ maximized when } p_2 = \frac{1}{N}$$

g) If p_1 is 0 (or p_1 is very small) then $P(\text{successful xmission by even device})$ can still be greater than 0, and thus, greater than $P(\text{successful xmission by odd device})$ *we always get priority to odd devices by waiting for them before sending.*



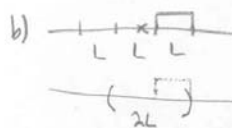
A device using slotted ALOHA will send on the next available slot if it has something to send. The next available slot can land anywhere in the $2L$ time window:



\therefore all slotted ALOHA "look" like regular ALOHA to any device using ALOHA

$$P(\text{successful transmission}) = e^{-n\lambda 2L} \quad \text{since there are effectively } n \text{ ALOHA devices}$$

(assuming a device can collide with itself)



$P(\text{successful transmission}) = P(\text{no slotted ALOHA device transmits in this length } L \text{ window})$
AND no ALOHA device falls within the $2L$ window shown to the left)

$$= e^{-(n-h)\lambda L} e^{-h\lambda 2L} = e^{-\lambda L(n+h)}$$

(assuming device can collide with itself)

$$\begin{aligned} c) P(\text{successful transmission}) &= P(\text{being ALOHA}) P(\text{success | ALOHA}) + P(\text{being slotted ALOHA}) P(\text{success | slotted ALOHA}) \\ &= \frac{h}{n} e^{-n\lambda 2L} + \frac{n-h}{n} e^{-\lambda L(n+h)} \end{aligned}$$

5) Once device i finishes transmitting, start at time 0 (memoryless property of exponential rv)

$$\begin{aligned} P(\text{being next device to transmit}) &= \frac{\lambda_2}{\lambda_1 + \lambda_2} \quad (\text{competing exponentials}) \\ &= \frac{1}{2} \quad \text{if } \lambda_1 = \lambda_2 \end{aligned}$$

b) $\frac{r \cdot C_1}{\# \text{ bits}} = \frac{-6+2-4}{8} = -1 \text{ by } C_1$

$$\frac{r \cdot C_2}{\# \text{ bits}} = -1 \text{ by } C_2$$

$$\frac{r \cdot C_3}{\# \text{ bits}} = -1 \text{ by } C_3$$