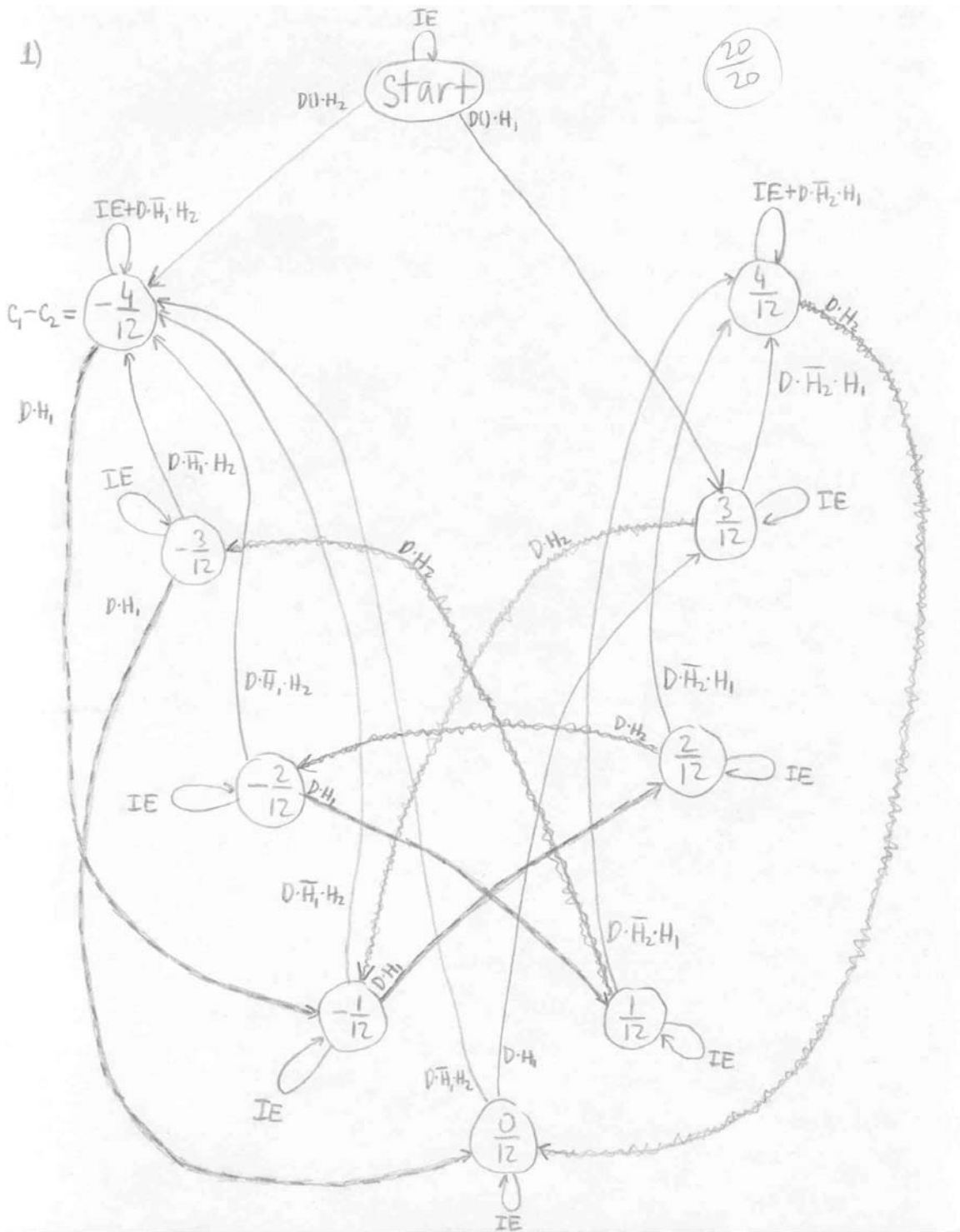


1)

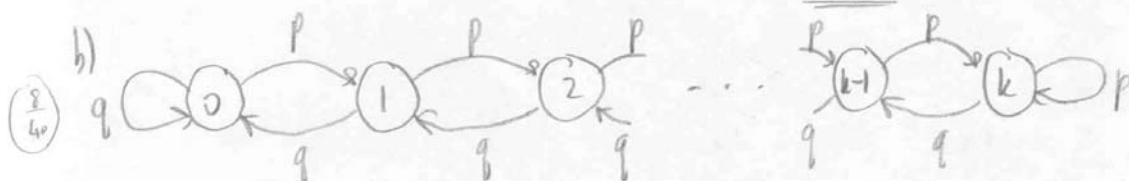
$\frac{20}{20}$



2) a) Let A be equal to 1 if the next event is an arrival, 0 otherwise
 $P\{A=1\} = \frac{\lambda}{\lambda+\mu} = p$ (whether or not the queue is empty, due to the pretend packets)

(8/40) $P\{S(n)=1 | S(n-1)=0\} = p = \frac{\lambda}{\lambda+\mu}$ for all n since the 0th state "loops back" on itself

Similarly, $P\{S(n)=0 | S(n-1)=0\} = 1-p = q = \frac{\mu}{\lambda+\mu}$ for all n



c) $\pi_i = \lim_{n \rightarrow \infty} P\{S(n)=i\}$, $P\{S(n)=i\} = P\{S(n-1)=i-1\}p + P\{S(n-1)=i+1\}q$ for $0 < i < k$

$P\{S(n)=i\} = P\{S(n-1)=i\}q + P\{S(n-1)=i+1\}p$ for $i=0$

$P\{S(n)=i\} = P\{S(n-1)=i-1\}p + P\{S(n-1)=i\}q$ for $i=k$

(8/40) $\pi_i = \lim_{n \rightarrow \infty} P\{S(n)=i\} = \lim_{n \rightarrow \infty} P\{S(n-1)=i\}$ as $n \rightarrow \infty$ $\pi_0 = \pi_0 q + \pi_1 q$ $\pi_0 - \pi_0 q = \pi_1 q$ $\pi_0 = \pi_1 q \frac{1}{1-q}$

($0 < i < k$) $\pi_i = \pi_{i-1}p + \pi_{i+1}q$ $\pi_{i+1} = \frac{\pi_i - \pi_{i-1}p}{q} = \pi_i \frac{p}{q}$

$\pi_k = \pi_{k-1}p + \pi_k p$ $\pi_k = \pi_{k-1}p \frac{1}{1-p}$
 $= \pi_{k-1} \frac{p}{q}$

Solve for all π_i in terms of π_0

$$\pi_1 = \pi_0 \frac{p}{q} \quad \pi_2 = \frac{\pi_1 - \pi_0 p}{q} = \frac{\pi_0 \frac{p}{q} - \pi_0 p}{q} = \pi_0 \frac{p - pq}{q^2} = \pi_0 \frac{p^2}{q^2}$$

$$\pi_3 = \frac{\pi_2 - \pi_1 p}{q} = \frac{1}{q} \left(\pi_0 \frac{p^2}{q^2} - \pi_0 \frac{p}{q} p \right) = \pi_0 \left(\frac{p^2}{q^3} - \frac{p^2}{q^2} \right) = \pi_0 \frac{p^3}{q^3} \dots$$

$$\pi_i = \pi_0 \left(\frac{p}{q} \right)^i \quad 0 < i \leq k$$

$$\sum_{i=0}^k \pi_i = 1 \quad \pi_0 + \sum_{i=1}^k \pi_i = 1 \quad \pi_0 + \sum_{i=1}^k \pi_0 \left(\frac{p}{q}\right)^i = 1 \quad \pi_0 \left(1 + \sum_{i=1}^k \left(\frac{p}{q}\right)^i\right) = 1$$

$$\pi_0 \left(1 + \frac{\frac{p}{q} - \left(\frac{p}{q}\right)^{k+1}}{1 - \frac{p}{q}}\right) = 1$$

$$\pi_0 = \frac{1}{1 + \left(\frac{\frac{p}{q} - \left(\frac{p}{q}\right)^{k+1}}{1 - \frac{p}{q}}\right)}$$

$$= \frac{1 - p/q}{1 - (p/q)^{k+1}}$$

$$\pi_i = \pi_0 \left(\frac{p}{q}\right)^i$$

$$\sum_{i=1}^k x^i = \dots = \frac{x - x^{k+1}}{1 - x}$$

d) $\pi_0, k \rightarrow \infty = (\text{if } p < q) \quad \frac{1}{1 + \frac{p/q}{1 - p/q}} = \frac{q-p}{q} = 1 - \frac{p}{q}$

and $\pi_i = \pi_0 \left(\frac{p}{q}\right)^i$

(8/40)

if $p \geq q$ $\pi_0 = \pi_i = 0$

(since as $k \rightarrow \infty$, $p > q$ will make the state drift to ∞)

e) It is due to the self-loop at the zeroth state,

while the problem in class had no self-loop at the zeroth state

(8/40)

3)

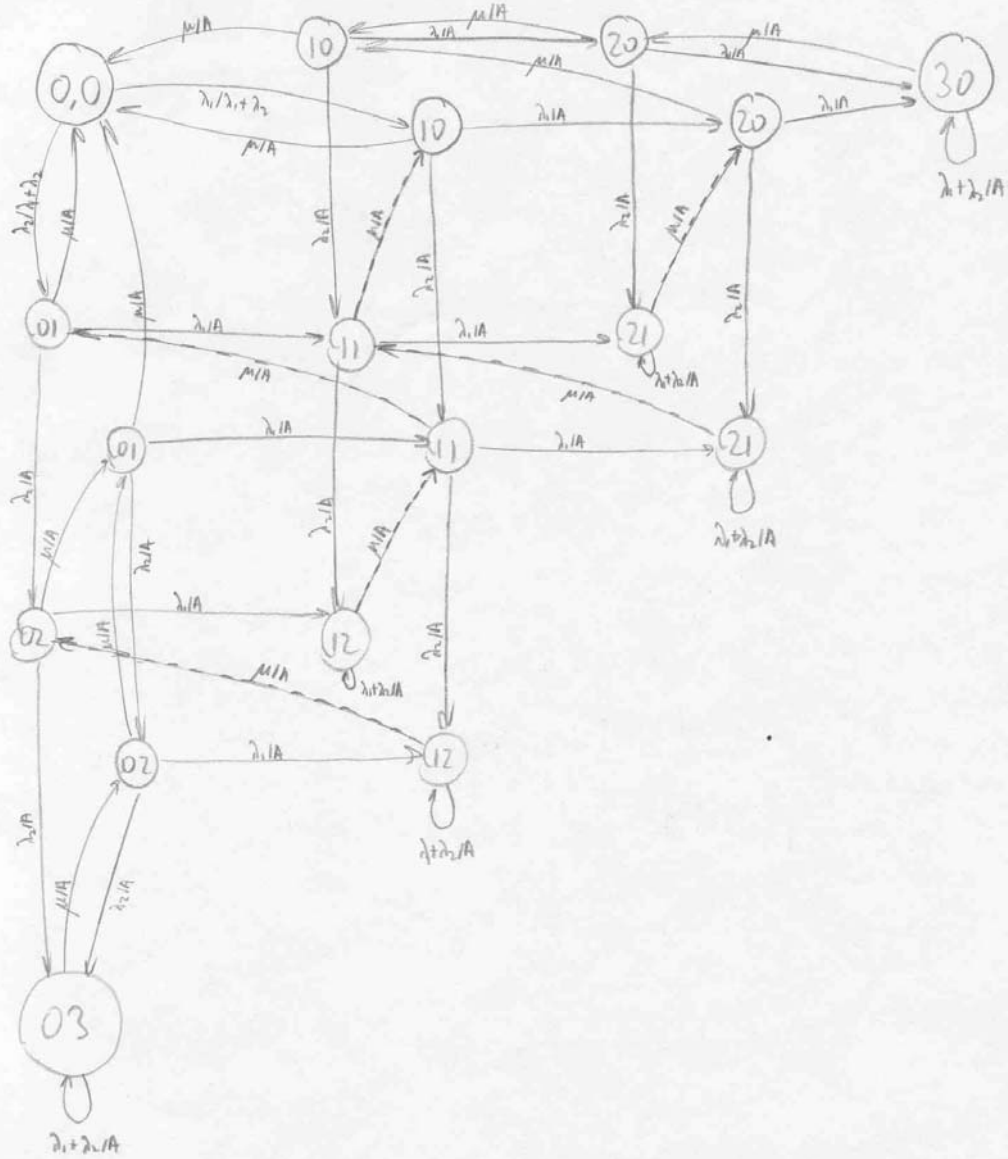
last processed f_1 : f_2 being processed

" f_2 : f_1 being processed

$$A = \lambda_1 + \lambda_2 + \mu$$

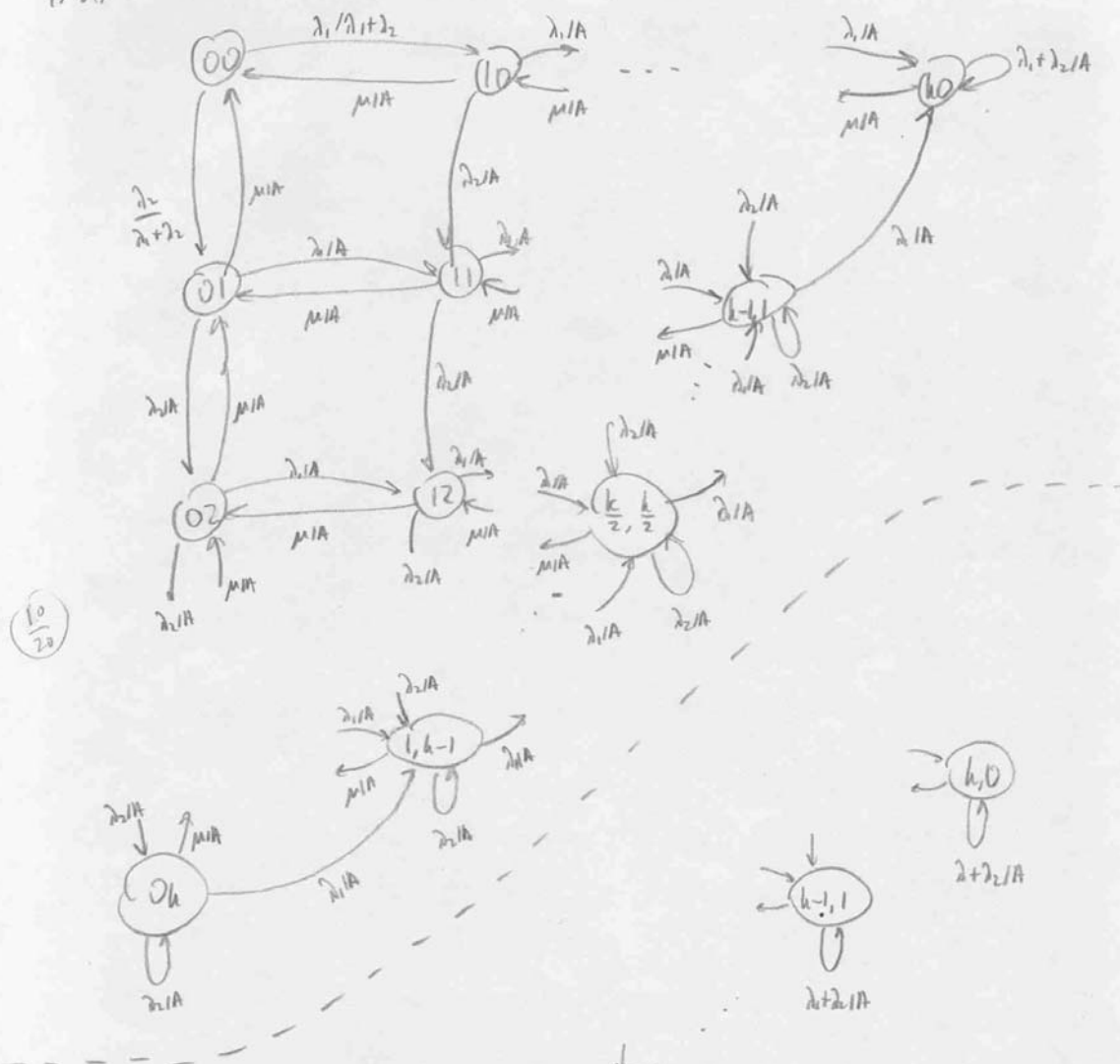
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x packets of f_1 y packets of f_2



4) a)

$$A \doteq \lambda_1 + \lambda_2 + \mu$$



$\frac{10}{20}$

b) Same as above, except on diagonal:

$\frac{10}{20}$

