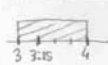


1) $N = \lambda T$ 50 shoppers = 10 shoppers/min $\cdot T$ $T = 5 \text{ mins} = \text{max avg. time customer spends in line}$
 but max. avg. time with Santa cannot be determined

2) $Y = \max\{X_1, \dots, X_k\}$ Let $Z_1 = \text{the first } X$
 $Z_2 = \text{the second } X - \text{the first } X$ (eg. $\begin{array}{ccc} x_1 & x_2 & x_3 \\ | & | & | \\ \hline & z_1 & z_2 \end{array}$) (remember: $E[\text{Exp}(\lambda) \text{ random var}] = \frac{1}{\lambda}$)
 $Z_3 = \text{the third } X - \text{the second } X$
 \vdots
 $E[Y] = E[Z_1 + Z_2 + \dots + Z_k] = E[Z_1] + E[Z_2] + \dots + E[Z_k] = \frac{1}{k\lambda} + \frac{1}{(k-1)\lambda} + \dots + \frac{1}{\lambda} = \frac{1}{\lambda} \sum_{x=0}^{k-1} \frac{1}{k-x} = \frac{1}{\lambda} \sum_{x=1}^k \frac{1}{x}$

3) a)  $P(\text{you missed both}) = P(\text{both uniform rvs \& land here}) = \frac{1}{4} \cdot \frac{1}{4}$

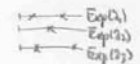
b) $P(\text{missed blue})P(\text{red within next 15 mins}) + P(\text{didn't miss blue})P(\text{either in next 15 mins})$
 $= \frac{1}{4} \cdot \frac{1}{2} + \frac{3}{4} \cdot (1 - \frac{1}{2}) = \frac{1}{4} + \frac{3}{4} \cdot \frac{1}{2} = \frac{1}{4} + \frac{3}{8} = \frac{5}{8}$
 this is true since $P(\text{red in next 15 mins} | \text{red hasn't come}) = \frac{P(\text{red in next 15 mins})}{P(\text{red hasn't come})} = \frac{1/4}{3/4} = \frac{1}{3}$

4) a) $P(\text{both before 15 mins}) = (1 - e^{-\lambda 15})(1 - e^{-\lambda 15})$ since $P(\text{Exp}(\lambda) \text{ random var} < x) = \int_0^x \lambda e^{-\lambda x'} dx' = 1 - e^{-\lambda x}$

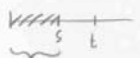
b) $P(\text{missed blue})P(\text{red within 15 mins}) + P(\text{didn't miss blue})P(\text{either in next 15 mins})$
 $= (1 - e^{-\lambda 15})(1 - e^{-\lambda 15}) + e^{-\lambda 15} \cdot (1 - e^{-2\lambda 15}) = 1 - 2e^{-\lambda 15} + e^{-2\lambda 15} + e^{-\lambda 15} - e^{-3\lambda 15}$
 $= 1 - e^{-\lambda 15} + e^{-2\lambda 15} - e^{-3\lambda 15}$
 due to memoryless property

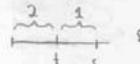
c) $P(\text{missed blue})P(\text{next is red}) + P(\text{didn't miss blue})P(\text{next is red})$
 $= (1 - e^{-\lambda 15}) \cdot \frac{1}{2} + e^{-\lambda 15} \cdot \frac{1}{2}$
 competing exponentials

5) a) $P(X_1 < X_2 < X_3) = P(X_1 < (X_2, X_3))P(X_2 < X_3 | X_1 < (X_2, X_3)) = P(X_1 < (X_2, X_3))P(X_2 < X_3)$ due to memoryless property
 where X_i is arrival time of bus i
 $= \left(\frac{\lambda_1}{\lambda_1 + (\lambda_2 + \lambda_3)}\right) \left(\frac{\lambda_2}{\lambda_2 + \lambda_3}\right)$

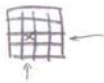
b)  $\Rightarrow \dots \text{Exp}(\lambda_1 + \lambda_2 + \lambda_3)$ $P(\text{no bus in time } t) = e^{-(\lambda_1 + \lambda_2 + \lambda_3)t}$ since $P(X > x) = e^{-\lambda x}$ for $X = \text{Exp}(\lambda)$

c) $P(\frac{1}{t}, \frac{2}{t})$ or $\frac{1}{t}, \frac{2}{t}$ or $\frac{2}{t}, \frac{1}{t}$ $= e^{-\lambda_1 t} (1 - e^{-\lambda_2 t})(1 - e^{-\lambda_3 t}) + e^{-\lambda_2 t} (1 - e^{-\lambda_1 t})(1 - e^{-\lambda_3 t}) + e^{-\lambda_3 t} (1 - e^{-\lambda_1 t})(1 - e^{-\lambda_2 t})$

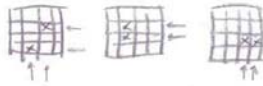
d)  as above, $P = (1 - e^{-\lambda_1 u})e^{-\lambda_2 u}e^{-\lambda_3 u} + (1 - e^{-\lambda_2 u})e^{-\lambda_1 u}e^{-\lambda_3 u} + (1 - e^{-\lambda_3 u})e^{-\lambda_1 u}e^{-\lambda_2 u}$
 due to memoryless property, set $s \rightarrow 0$, $t \rightarrow t-s \triangleq u$ where $u = t-s$

e)  since $\int_t^s \lambda e^{-\lambda x} dx = -e^{-\lambda x} \Big|_t^s = -e^{-\lambda s} + e^{-\lambda t} = e^{-\lambda t} - e^{-\lambda s}$
 we have $P(\text{exactly 2 before } t | \text{all before } s) = \frac{P(\text{exactly 2 before } t, \text{all before } s)}{P(\text{all before } s)}$
 $= \frac{(1 - e^{-\lambda_1 t})(1 - e^{-\lambda_2 t})(e^{-\lambda_3 t} - e^{-\lambda_3 s}) + (1 - e^{-\lambda_1 t})(e^{-\lambda_3 t} - e^{-\lambda_3 s})(e^{-\lambda_2 t} - e^{-\lambda_2 s}) + (1 - e^{-\lambda_2 t})(e^{-\lambda_3 t} - e^{-\lambda_3 s})(e^{-\lambda_1 t} - e^{-\lambda_1 s})}{(1 - e^{-\lambda_1 s})(1 - e^{-\lambda_2 s})(1 - e^{-\lambda_3 s})}$

6) a)



all 1-bit errors detectable from row & column parity checks



all 2-bit errors detectable from row or column parity checks



all 3-bit errors detectable from row & column parity checks

b)



detectable



not detectable

c)



start with this



reorder the rows - still undetectable



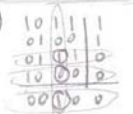
then choose the columns - still undetectable

(remember: # ways of picking 3 rows out of $k+1$ rows where order does not matter = $\binom{k+1}{3} = \frac{(k+1)!}{3!(k+1-3)!}$
where order does matter = $\frac{(k+1)!}{(k+1-3)!}$)

(if you don't see why the cols are chosen instead of reordered, try drawing out the case where $k=2$)

$$P = p^6 (1-p)^{\binom{k+1}{3} - 6} \cdot \frac{(k+1)!}{(k+1-3)!} \cdot \binom{k+1}{3}$$

7)



more than 2 bit error

8) a)

$$\begin{pmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

$\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ is 3rd column, so (101100) must likely be missed

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \oplus \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad \therefore \text{1st and 2nd bits most likely flipped}$$

c) no 3 cols add to $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$, no 4 cols add to $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$, so no 8 no

$$\uparrow \begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \end{pmatrix}$$

a) since all cols are non-zero, the syndrome $S = Cw_r$ will always be non-zero when the error vector contains a single error. Thus all single errors are detectable.

$$b) \frac{x}{1} = \frac{x}{2} = \frac{x}{4} = \frac{x}{5} = \frac{x}{6} = \frac{x}{7} \quad \text{errors in 1st, 4th, 5th, 6th bits}$$