HW Solutions #7

ELEN E4710 - Intro to Network Engineering — Due 12/13/2004
Fall 2004 — Prof. Rubenstein

Homework must be turned in at the beginning of class on the due date indicated above. CVN students have one additional day. Late assignments will not be accepted.

1. A sender S and receiver R use the Selective-Repeat protocol to communicate reliably across a network that drops or corrupts packets in either direction (e.g., from S to R or from R to S) with probability p. Assume that the loss process is Bernoulli and that the sender never times out early (i.e., the sender does not retransmit a packet while a previous copy is heading to the receiver using the Selective-Repeat protocol or an acknowledgment for a previous transmission is on its way back to the sender).

   (a) Compute the expected number of times the sender transmits a particular packet (i.e., packet i for any i). Ans)
   Define T to be the number of times the sender transmits a particular packet.
   $E[T] = \sum_{i=0}^{\infty} P\{T > i\} = \sum_{i=0}^{\infty} p^i = 1/(1-p)$. ∎

   (b) If the window size used by the protocol is w, and it takes time τ for a packet to be received and acknowledged when no loss or corruptions take place, give an upper bound on the maximum rate of the protocol. Explain why your result is an upper bound (hint: *at most* w packets can be transmitted at any given time). Ans)
   By Little's Law, the average rate equals the average number of packets in the system divided by the average time spent in the system for each packet. Therefore, the upper bound of maximum rate is w/τ. ∎

2. Assume S and R communicate in a networking environment similar to that in problem 1. The reliable data transfer protocol used is Go-back-N, where packets that are received out of order are dropped by the receiver (i.e., packet $i+1$ is accepted only if packet i has already been received). Assume the sender and receiver communicate in rounds where each round, the sender sends the w packets that are currently in its window.

   (a) Assuming ACKs are never lost, compute the expected number of packets accepted by the receiver each round. Ans)
   Define $X_i = 1$ if ith packet in window is accepted and $X_i = 0$ otherwise.
   $P\{X_i = 1\} = (1-p)^i$. $E[\sum_{i=1}^{w} X_i] = \sum_{i=1}^{w} E[X_i] = \sum_{i=1}^{w} (1-p)^i = \frac{(1-p)^w - 1}{(1-p)-1}$.
   ∎

   (b) Suppose the receiver responds each round by sending a single ACK (at the end of the round) indicating the largest sequence number it has accepted, and that this ACK is lost with probability p. Let N be a r.v. that equals the number of packets accepted by the receiver between the time the sender receives acknowledgments. Let R be the number of rounds that take place until the sender receives an acknowledgment. Compute $E[R]$ and $E[N]$. Ans)
   Consider the ACK as a particular packet, we get the same answer for $E[R]$ as which in problem 1)a. $E[R] = 1/(1-p)$. Specifically, $P\{R = r\} = p^{r-1}(1-p)$.
   Suppose we know it takes r round for getting the ACK, the number of packets received has the following upper and lower bounds:
   upper bound: $p_u(r) = \sum_{i=1}^{w} (1-p^r)^i$ lower bound: $p_l(r) = \sum_{i=1}^{w} 1 - [1-(1-p)^i]^r$
   Therefore, by conditioning on the number of rounds to receive the ACK, we have the bounds of the expected number of received packets $E[N]$ to be:
   $E_u[N] = \sum_{r=1}^{\infty} p_u(r) P\{R = r\} = \sum_{r=1}^{\infty} \sum_{i=1}^{w} (1-p^r)^i p^{r-1}(1-p)$.
   $E_l[N] = \sum_{r=1}^{\infty} p_l(r) P\{R = r\} = \sum_{r=1}^{\infty} \sum_{i=1}^{w} 1 - [1-(1-p)^i]^r p^{r-1}(1-p)$.
   ∎

(c) (Extra Credit) The expected number of transmissions per round is $E[N]/E[R]$ (and not $E[N/R]$). Why? Ans)

In a long run, the average performance of the protocol is

$$\lim_{l\to\infty} \frac{\sum_{i=1}^{l} N_i/l}{\sum_{i=1}^{l} R_i/l} = \frac{\lim_{l\to\infty} \sum_{i=1}^{l} N_i/l}{\lim_{l\to\infty} \sum_{i=1}^{l} R_i/l} = E[N]/E[R].$$

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3. Consider a reliable data transfer protocol that transfers data using a window of size w in an environment in which packets can be lost, but cannot be reordered. The protocol uses a sequence numbering scheme that goes from 0 to $n-1$, such that packet representing the ith segment of data is assigned sequence number $i \pmod n$. If n is too small, the receiver might mistake one data packet for another. For instance, if $n < w$, then the receiver might mistake packet n (which has sequence number 0) as a retransmission of packet 0, which also has sequence number 0, as both packets could be in the window at the same time. Show that a receiver can make such an error when

(a) The protocol is Selective Repeat and $n = 2w - 1$. Ans)

Suppose the receiver has successfully received packets $0, 1, \ldots, w-1$. The receiving window is expecting packets $w, w+1, \ldots, 2w-2, 0', \ldots$. But ACK 0 is lost. When the sender retransmits packet 0, the receiver takes it as packet $0'$. Because they have the same sequence number.

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(b) Go-Back-N (assume the receiver discards any packets that cause a gap in the received sequence) and $n = w$. Ans)

Suppose the receiver has successfully received packets $0, 1, \ldots, w-1$, but all ACKs are lost. When the sender retransmits packet 0, the receiver will take it as the wth packet, which has the same sequence number as packet 0.

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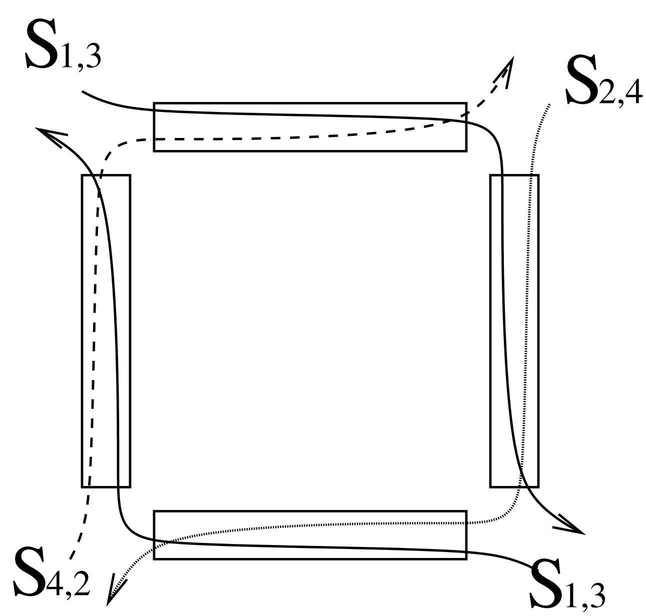

4. Consider the network pictured above where each link has capacity C, each session transmits into the network at rate ρ, and each session's transmissions traverse two links. In class, we showed how increasing the flow rate beyond $C/2$ led to congestion collapse. How can priority queueing be used to prevent collapse where the priority mechanism does not know the capacity of the link (i.e., you cannot simply restrict the entry rate of the flow into the link to $C/2$)? (Hint: How should each link prioritize the session data that it carries so that flow is not "wasted" by passing through the first link and then being dropped at the second?)

   Ans) Each link has two incoming flows. We assign a higher priority to the flow which departs from a previous link.

   To show there is no collapse, let λ be the rate at which the flows transmit, $\lambda > C/2$.

   Let $\lambda^{'}$ be the rate the flow comes out of the first link. Since this flow has priority going into the next link, $\lambda^{'}$ will also be the rate at which the flow exits the 2nd link.

   Hence, $\lambda + \lambda^{'}$ is the rate going in, and $2\lambda^{'}$ is the rate coming out. Thus, $\lambda^{'} = C/2$.

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5. A flow will traverse a set of routers $R_1, \cdots, R_n$ where router R_i will process its packets at (deterministic) rate λ_i and can queue up to b_i packets at a time without dropping any.

   (a) Assuming the flow has reserved access to the router resources (such that no other flow competes for the resources described above), what is the leaky bucket configuration (ρ, b) that the flow should use (where ρ is the rate at which tokens enter the bucket and b is the maximum number of tokens the bucket can hold) to maximize the transmission rate such that a packet is never lost?
   Ans) $\rho = min\{\lambda_1, \lambda_2, \ldots, \lambda_n\}; b = b_1$.
   ρ is the maximum achievable transmission rate.
   b is the maximum number of burst packets allowed. Obviously, we want $b \leq b1$ so that no packet will be dropped at router 1. Since we set ρ to be the minimum processing rate of the router, the arrival rate of each router will be less than or equal to its processing rate. So no packet overflow will occur at any subsequent router.

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   (b) Give an example of transmission sequence that leads to a packet loss when the leaky bucket is configured using $(\rho^{'}, b^{'})$, where $\rho^{'} = \rho$ and $b^{'} > b$.
   Ans)
   Packets come in a burst at rate $b^{'}$. Packets get dropped at router r_1.

   ∎

   (c) Give an example of transmission sequence that leads to a packet loss when the leaky bucket is configured using $(\rho^{'}, b^{'})$, where $\rho^{'} > \rho$ and $b^{'} = b$.
   Ans)
   Suppose $\rho^{'} > \rho = \rho_1$. Packets get dropped at router r_1.

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