1. Consider the networking system depicted above where each link has capacity $C$ and each flow traverses two hops, with each source transmitting data at the same rate, $\rho > C/2$. Assume that each router is bufferless (i.e., if a packet comes when another packet is being processed in the router’s buffer, it must be dropped), and that the router uses virtual clock to schedule packets. Using a fluid model, where $\rho$ is the rate at which each flow transmits into the network, let $\gamma$ be the rate at which a single flow exits the first hop of the transmission, and let $\lambda$ be the rate at which flow exits the second hop. Let $A_\rho$ be the fraction of first-hop flow that is accepted (i.e., each flow is rejected from the first hop of the network at rate $(1 - A_\rho) \rho$) and let $A_\gamma$ be the fraction of second-hop flow that is accepted.

(a) Give the equation for $A_\rho$ as a function of $\rho$, $\gamma$, $A_\rho$, $A_\gamma$.

(b) Give equations for

i. $\gamma$ as a function of $\rho$ and $A_\rho$.
ii. $\lambda$ as a function of $\gamma$ and $A_\gamma$.

Not all variables listed need to appear in the equation. The equations in this part should not depend on $C$.

(c) Explain why we cannot have $A_\gamma \gamma > A_\rho \rho$. (Hint: this holds true even without virtual clock, so you do not need to use the fact that the routing uses virtual clock here).

(d) Prove that we cannot have $A_\gamma \gamma < A_\rho \rho$.

(e) Since we must have $A_\gamma \gamma = A_\rho \rho$, use the formulas computed in parts 1a and 1b to compute $\lambda$.

(f) Using the results above, indicate whether congestion collapse can occur in the above system when flows transmit at a constant rate $\rho$ and virtual clock is the queueing discipline applied.
2. Consider a flow that sends packets at a fixed rate, $\alpha$ through a router that applies a leaky bucket congestion control mechanism, where tokens enter the leaky bucket at rate $\rho$, and the bucket has size $b$. The flow bundles together $k$ packets and sends these packets to the router every $s = \alpha/k$ seconds.

(a) In terms of $s, \rho, b, \alpha$ and $k$, what is the long-term rate of the flow? More precisely, let $n(t)$ be the number of packets that have passed through the router by time $t$, where $n(0) = 0$ and the flow starts transmitting at time $t = 0$. What is $\lim_{t \to \infty} n(t)/t$?

(b) Prove that setting $k = 1$ (and hence $s = \alpha$) achieves the highest possible rate.

(c) It is likely that this highest possible rate can also be achieved for other values of $k$. What is the maximum value of $k$, where $s = \alpha/k$ that achieves the same rate through the router?

3. In a weighted max-min fair allocation, each flow $f_i$ is assigned a weight $w_i$ such that an allocation of rates $(\rho_1, \cdots, \rho_n)$ to flows $f_1, \cdots, f_n$ is weighted max-min fair whenever the allocation is feasible and if, for any $\epsilon$, any flow $f_i$’s rate is increased to $\rho_i + \epsilon$, to produce a feasible allocation, another flow $f_j$’s rate must be decreased where $\rho_j/w_j \leq \rho_i/w_i$. Note that when all weights are set to identical values, the weighted max-min fair allocation equals the max-min fair allocation.

(a) Consider three flows $f_1, f_2, f_3$ that are assigned weights 1,3,6 respectively. If the network consists of a single link with capacity 10 and all three flows traverse this link, what are the respective rates within the weighted max-min fair allocation?

(b) In the figure above, if all flow weights are 1, what is the max-min fair allocation?

(c) In the figure above, the flow weights are indicated in parentheses next to the source of the flow. What is the weighted max-min fair allocation in this network?

(d) Give a one-sentence description of how the algorithm that computes the max-min fair allocation needs to be modified to produce the weighted max-min fair allocation.