1. Let \( G \) be an undirected graph with \( n \) nodes and \( m \) edges in which no two edges are assigned the same weight. Let \( r \) be a node arbitrarily selected in \( G \).

   (a) Prove that any such \( G \) has a unique MST (Hint: the proof can be done using a part of the proof covered in class that proved the correctness of the MST algorithm).

   (b) Construct a graph \( G \) where there are multiple shortest path trees rooted at \( r \).

2. Compute an MST for the above graph using either algorithm covered in class. Write down the order in which edges are added.

3. Compute a shortest path tree rooted at node \( A \).

   (a) using Dijkstra’s algorithm.

   (b) using the Bellman-Ford algorithm (include the information about the predecessor node).

   (c) Assume that after the Bellman-Ford algorithm completes (i.e., no further changes are made to the tree), the weight of edge \((A, C)\) changes to 1. Continue the algorithm to find the new shortest path.

   (d) Assume that after the Bellman-Ford algorithm completes for a second time, edge \((C, D)\) changes its weight to 10. Continue the algorithm once more to find the new shortest path.
4. Consider the two graphs drawn above.

(a) Show, by changing a single edge weight, how without Poison Reverse, the graph on the left exhibits the count-to-infinity problem.

(b) Show, by changing the same edge weight, how with Poison Reverse, the graph on the left no longer exhibits the count-to-infinity problem.

(c) Show, by changing a single edge weight in the graph on the right, how with Poison Reverse, the graph still exhibits the count-to-infinity problem.

5. In the LAN above, switch IDs are indicated upon the switch, and wire weights, assigned by a network administrator, are depicted adjacent to the wire.

(a) Perform the spanning tree algorithm to determine the interfaces (ports) of the switches that are turned on for forwarding.

(b) Suppose the switch with ID 22 is removed. Redraw the spanning tree after this modification.

(c) Suppose the switch with ID 2 is removed from the original configuration. Redraw the spanning tree now.