1. In the bridged LAN pictured above, $k$ LANs are bridged together using switches. Each LAN contains $n$ devices such that there are $kn$ devices on the bridged LAN. The devices participate in a slotted protocol, with each device transmitting a frame in a given slot with probability $p$, the destination location is uniformly distributed among the other devices.

   (a) An additional device $D$ is added to LAN 1 (so that there are $n+1$ devices on this LAN). If $X_i$ is a random variable that equals 1 if device $D$ tries to transmit during the $i$th slot, and $S_i$ is a random variable that equals 1 when $D$'s transmission during the $i$th slot is successful, what is $Pr(S_i = 1|X_i = 1)$?

   (b) Suppose device $D$ reduces the probability of transmission to $p/n$. How does this affect $Pr(S_i = 1|X_i = 1)$?

2. $N + 1$ devices numbered 1, $\cdots$, $N + 1$ share a LAN and frame transmission times are segmented into slots such that two transmissions during the same slot always cause transmission failures, whereas two transmissions during different slots do not cause transmission failures. Odd numbered slots are used to send new transmissions. If the transmission fails because of a collision, then the subsequent even slot is used to attempt a retransmission. The probability that a device transmits on an odd-numbered slot is $p$, where the decision to transmit is independent from the device's previous transmission history and from the transmission decisions of the other devices. Given the device's transmission in an odd slot collides with other transmissions, the device reattempts the transmission during the even round with probability $q$. The decision to retransmit during the even round is independent of the decisions by the other devices that also experienced a collision in the previous odd round.

   Let $X_{i,j}$ be an indicator random variable that equals one if device $i$ attempts a transmission during round $2j + 1$ and 0 otherwise. Let $Y_{i,j}$ be an indicator random variable that equals 1 if device $i$ succeeds in completing its transmission (if one is made) in either round $2j + 1$ or in round $2j + 2$.

   What is $P(Y_{i,0} = 1|X_{i,0} = 1)$?

3. Suppose that $n$ devices share a LAN, where each device sends frames that take $L$ microseconds to transmit onto the wire, where $\tau$ is the maximum propagation delay on the LAN. $k$ of these $n$ devices use ALOHA (without carrier sensing), where the backoff occurs at rate $\lambda$, the other $n - k$ devices use slotted ALOHA with slots of size $L + \tau$ and backoff at rate $\lambda$. What is the probability of successful transmission for
(a) A device using ALOHA
(b) A device using slotted ALOHA
(c) A device drawn at uniformly at random from the set of devices.

4. Device A and Device B share a common LAN on which they transmit. They can sense each other’s signals in time $\tau > 0$. After a device successfully transmits, it backs off before its next transmission for a time exponentially distributed with rate $\lambda_1$. When an attempted transmission fails, it backs off for an exponentially distributed time with rate $\lambda_2$ before attempting a retransmission. Assume that when collisions occur,

(a) Explain why, if $\lambda_1 = \lambda_2$, the device that just completed transmission is more likely to be the next device to retransmit (hint: this would not be the case if $\tau = 0$).
(b) What should $\lambda_1$ equal, in terms of $\lambda_2$ and $\tau$ so that the likelihood of the next transmission is equalized among the two devices?

5. Let $(1, 1, -1, 1, -1, 1, -1)$ and $(1, -1, 1, 1, 1, 1, 1)$ be two chirping sequences used by devices communicating with the same base station.

(a) Show that both of these chirping sequences can be used simultaneously.
(b) Can a third chirping sequence be constructed that can be used along with these other two? If so, give such a sequence. If not, prove why.