HW #2

ELEN E4710 - Intro to Network Engineering — Due 9/24/03
Fall 2003 — Prof. Rubenstein

Homework must be turned in at the beginning of class on the due date indicated above. CVN students have one additional day. Late assignments will not be accepted.

For each problem, you should convert the written question into a well-posed mathematical statement (i.e., construct the appropriate random variables and pose the problem using mathematical notation that involves those random variables). Only then should you proceed to solve the question (in terms of the variables defined in the mathematical statement).

1. Air conditioners turn on in the summertime at a rate of 50 a second, and stay on for an average of 7 hours. What is the expected number of air conditioners on at any given time?

2. n kernels of popping corn are placed in a pot and heated up. The time it takes for each kernel to pop is exponentially distributed with rate λ. A particular kernel is tagged, and T is a random variable that equals the time between the tagged kernel popping and the next kernel popping. What is $P(T > t)$? (Hint: consider the cases separately where the tagged kernel is the kth kernel to pop for all $1 \leq k \leq n$.

3. You arrive at a bus stop at 3:15 pm. Two busses, one red, one blue are both scheduled to arrive independently according to a uniform distribution between 3 and 4 pm.

   (a) What is the probability that the red bus comes first?

   (b) Somebody informs you that the red bus has not yet arrived, but is unsure about the blue bus. Given this information, what is the probability that the red bus arrives at least 15 minutes before the blue bus?

4. You arrive at a bus stop at 3:15 pm. Two busses, one red, one blue, arrive according to an exponential distribution with rate λ, starting at 3pm (they will not arrive before then).

   (a) What is the probability that the red bus comes first.

   (b) Somebody informs you that the red bus has not yet arrived. Given this information, what is the probability that the red bus arrives at least 15 minutes before the blue bus?

   (c) Suppose the red bus arrives at rate μ and the blue bus arrives at rate λ. Given the same information as in part (4b), what is the probability that the next bus to arrive is red?

5. Suppose that the arrival times of three buses numbered 1,2,3 are exponentially distributed with rates λ_1, λ_2, and λ_3 (here, the any bus can come first, i.e., the clocks for all buses start at time 0).

   (a) What is the probability that buses arrive in the order 1,2,3?

   (b) What is the probability that no bus arrives within time t?

   (c) What is the probability that exactly 2 buses arrive before time t?

   (d) What is the probability that at least 1 bus arrives before time t, given that no buses arrive before time $s < t$?

   (e) What is the probability that at least 2 buses arrive before time t, given that all buses arrive before time $s > t$?

6. Consider a 2-D parity check code.

   (a) Prove that any combination of 1, 2, or 3 bit errors is detectable.

   (b) Suppose the following correction procedure is used to correct detected bit errors. Let S_i be the set of original codewords, which, after i bits are flipped, produce the received codeword. Choose the smallest i where $|S_i| \neq 0$. Choose some correct codeword in this S_i uniformly at random and return that result.

   Let C be an indicator r.v. that equals 1 if the returned codeword is correct, and is 0 otherwise, and let X be a random variable that equals the number of bits flipped in the transmitted codeword, where each bit (including the checkbits) is flipped independently with probability p.

   What is $P(C = 0|X \leq 3)$?

$$
\begin{array}{cccc|c}
1 & 0 & 1 & 1 & 1 \\
0 & 1 & 0 & 0 & 0 \\
0 & 1 & 1 & 0 & 0 \\
1 & 0 & 0 & 1 & 0 \\
\hline
1 & 0 & 0 & 0 & 1
\end{array}
$$

7. Fix the most likely set of bit errors in the above codeword, under the assumption that bits are flipped with a very low probability.

8. Suppose you receive the codeword 1001100 which was generated using the (7,4) Linear code whose code bits are generated as follows:

   - $c_1 = b_1 \oplus b_3 \oplus b_4$
   - $c_2 = b_1 \oplus b_2 \oplus b_4$
   - $c_3 = b_1 \oplus b_2 \oplus b_3$

   (a) What codeword was most likely transmitted?

   (b) Suppose you are told that the error bits are correct, but that 2 data bits are flipped. Which bits are most likely flipped (there may be several options)?

   (c) Suppose the error bits are correct. Can 3 data bits be flipped? What about 4? Explain how you know this.

9. Suppose we switch to the linear code:

   - $c_1 = b_1 \oplus b_3$
   - $c_2 = b_2 \oplus b_4$
   - $c_3 = b_2 \oplus b_3$

   (a) Can this code always detect single-bit errors? Explain why or why not.

   (b) List the sets of single-bit errors that should be detected, but not repaired (because 2 possible repairs are equally likely).