HW #1

ELEN E4710 - Intro to Network Engineering Due 9/15/2003
Fall 2003 Prof. Rubenstein

Homework must be turned in at the beginning of class on the due date indicated above. CVN students have one additional day. Late assignments will not be accepted.

For each problem, you should convert the written question into a well-posed mathematical statement (i.e., construct the appropriate random variables and pose the problem using mathematical notation that involves those random variables). Only then should you proceed to solve the question (in terms of the variables defined in the mathematical statement).

1. What is the expected number of times that a fair n-sided die (each side is equally likely to come up) will be rolled
   (a) until the value of one is rolled?
   (b) until the value of the first roll is repeated?
   (c) until some value rolled previously is repeated?

2. A fair coin is tossed 10 times. What is the probability that either the either the first and tenth toss are the same, or the first and second toss are both tails?

3. A Casino offers the game Double-or-Half Roullette. A player places k dollars on the table and spins the wheel that contains 25 positions, numbered 1 through m. If the ball lands in a position whose number is larger than $n \leq m$, the player receives an additional k dollars (their money doubles). Otherwise, the player loses $k/2$ dollars (their money is halved). If the Casino wants the odds to be in its favor (such that the expected money won by a contestant per play is less than 0), how large must n be?

4. Consider a drawer containing n red balls and $m \leq n$ blue balls.
   (a) Suppose k balls are selected with replacement (after choosing a ball, it is returned to the pool and can be selected again). Let X be the number of red balls selected and Y the number of blue balls such that $X + Y = k$. What is $P(X < Y)$?
   (b) Suppose the balls are selected without replacement. What is $P(X < Y)$ for $k < m$?
   (c) For the case where the ball is not returned after each selection, let W be the number of times that the ith pick and the $i - 1$st pick are the same color. What is $E[W]$? (Hint: sum of expectations equaling expectation of sums is probably very useful here).

5. Let X, Y, and Z be random variables such that X and Y are dependent, and X and Z are dependent. Even so, Y and Z can be independent. Give an example. (Hint: think about the sum of 2 dice)

6. Pickpockets often work in teams of two where one thief (acting as the decoy) bumps into the target in an effort to distract the target - the other then grabs the wallet from the distracted target. In a crowded subway, there is a probability p_1 that during your trip, someone will bump into you. With probability p_2, this bump is part of a pickpocket ploy. With probability p_3, you catch on that you've been pickpocketed. With probability p_4, you catch the theif that takes your wallet.

   Let B be a random variable that equals 1 if you are bumped, and is 0 otherwise, $P(B = 1) = p_1$. L is a random variable that equals 1 if your wallet is lifted, and is 0 otherwise, $P(L = 1) = p_2$. D is a random variable that equals 1 if you detect the lifting of your wallet, and is 0 otherwise: $P(D = 1|B = 1, L = 1) = p_3, P(D = 1|B = 0) = 0, P(D = 1|L = 0) = 0$. Let X be a random variable that equals 1 if you catch the thief and equals 0 otherwise, $P(X = 1|D = 1) = p_4, P(X = 1|D = 0) = 0$.

   (a) What is $P(X = 1)$?
   (b) Are B and D independent? Explain (via mathematical notation).
   (c) What is $P(D = 0|B = 1)$?
   (d) What is $P(X = 0|B = 1)$?