HW #7

COMS W4119 - Computer Networks — Due Apr 12, 2006
Spring 2006 — Prof. Rubenstein

Homework must be turned in at the beginning of class on the due date indicated above. CVN students have one additional day. Late assignments will not be accepted.

1. Consider a 2-D parity check code.
   (a) Prove that any combination of 1, 2, or 3 bit errors is detectable.
   (b) For a 16-bit data word (i.e., code bits are extra bits), show a 4 bit error combination that cannot be detected, and show one that can.
   (c) For a data word with k^2 bits, given that bit flips are a Bernoulli process (independent) with probability p, compute the probability that 4 bit-errors occur and that the 2-D parity check fails to detect that the word is corrupted.

$$
\begin{array}{cccc|c}
1 & 0 & 1 & 1 & 1 \\
0 & 1 & 0 & 0 & 1 \\
1 & 1 & 1 & 1 & 0 \\
1 & 0 & 1 & 0 & 1 \\
\hline
1 & 0 & 0 & 0 & 1
\end{array}
$$

   (d) Fix the most likely set of bit errors in the above codeword.

2. Consider the (7,4) Linear code whose code bits are generated as follows:
   - $c_1 = b_1 \oplus b_3 \oplus b_4$
   - $c_2 = b_1 \oplus b_2 \oplus b_4$
   - $c_3 = b_2 \oplus b_3 \oplus b_4$

   Suppose you receive the codeword 1110111 which was generated using the above linear code. What codeword was most likely transmitted? Use the check matrix to obtain your result (you may also want to verify your resulting codeword is valid).

3. The above codes work best when p, the probability of a bit being flipped, is very small. What if p were very large (e.g., $1 - \epsilon$ for some very small ϵ) and Bernoulli. How would you modify the coding technique to get guarantees that were as good as if p were very small (i.e., $p = \epsilon$)?

4. Suppose we switch to the linear code:
   - $c_1 = b_1 \oplus b_3$
   - $c_2 = b_2 \oplus b_4$
   - $c_3 = b_2 \oplus b_3$

   (a) Can this code always detect single-bit errors? Explain why or why not via the check matrix.
   (b) List the sets of single-bit errors that should be detected, but not repaired (because 2 possible repairs are equally likely).
   (c) What is the Hamming Distance of this code? Explain why using the results from above. Give two valid codewords whose Hamming Distance equals the Hamming Distance of the code.

5. Suppose that n devices share a transmission medium, where each device sends frames that take L microseconds to transmit onto the wire. k of these n devices use ALOHA, where the backoff occurs at rate λ, the other $n - k$ devices use slotted ALOHA with slots of size L and backoff at rate λ. What is the probability of successful transmission for

   (a) A device using ALOHA (w/o carrier sensing)

   (b) A device using slotted ALOHA

6. N devices share a transmission medium and frame transmission times are segmented into slots such that two transmissions during the same slot always cause transmission failures, whereas two transmissions during different slots do not cause transmission failures. Assume that each device has a frame to send every time-slot, but only performs the transmission with a probability p.

   (a) Suppose this slotted collision avoidance mechanism is implemented using slotted ALOHA, where slots last for time T. What is the rate of the backoff timer as a function of T and p?

   (b) What is the probability of a successful transmission in a given timetick (in terms of N and p).

   (c) What is the probability that device 1's transmission is successful, given device 1 attempts a transmission?

   (d) What is the expected number of successful transmissions per time-slot?

   (e) What value of p (in terms of N) maximizes the probability in part 6b.

7. Let C_1, C_2, C_3 be three connections that use CDMA to transmit upon the same channel using chipping signals (1,1,1,1,1,1,1,1), (1,1,-1,-1,1,1,-1,-1), and (1,1,-1,-1,-1,-1,1,1) respectively.

   (a) If the received chipping signal is (1,1,1,1,-1,-1,3,3), what was the value of the bit transmitted by connections C_1, C_2, C_3 (where the value is either 1 or -1)?

   (b) Derive a fourth coding signal that is orthogonal to all three codes above.