HW #7 SOLUTION

COMS W4119 – Computer Networks
Spring 2006

PROBLEM 1

(a)
Any one-bit error is detectable because wherever the error occurs, the corresponding row and column parities will not match.

Any two bit error is detectable as follows:

- If the bits lie on different rows and columns, then 2 row parities and 2 column parities will not match
- If the bits lie on the same row, then the corresponding row parity might or might not match, but the 2 corresponding column parities will not match
- If the bits lie on the same column, then the corresponding column parity might or might not match, but the 2 corresponding row parities will not match

Any three bit error is detectable as follows:

- If all 3 bits lie on the same row or column, then the parity for the corresponding row or column will not match
- In every other case there must be a single error bit that lies on a separate row/column, therefore the parity for that row/column will not match.

(b)
Detectable error:

Original Transmission:

$$\begin{array}{cccc|c} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 \end{array}$$

Noisy message received:

$$\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 \end{array}$$

Non-Detectable error:

Original Transmission:

$$\begin{array}{cccc|c} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 \end{array}$$

Noisy message received:

$$\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 \end{array}$$

(c)

We keep in mind that, since the data word has k^2 bits, the transmitted block is (k+1) by (k+1) after adding the parity bits.

P(4 bit error detected | 4 bit error occurred)
  = P(4 bit error occurred but not detected) / P(4 bit error occurred)

But P(4 bit error occurred but not detected) = $((k+1) \text{ choose } 4)^2\ p^4\ ((1-p)^{((k+1)(k+1)-4)}$
Since we need to choose 2 out of k+1 rows and 2 out of k+1 columns where the bit flips will occur.

And P(4 bit error occurred) = $((k+1)^2 \text{ choose } 4)\ p^4\ (1-p)^{((k+1)(k+1)-4)}$

➔ P(4 bit error detected | 4 bit error occurred) = $((k+1) \text{ choose } 4)^2 / ((k+1)^2 \text{ choose } 4)$

(d)

$$
\begin{array}{cccc|c}
1 & 0 & 1 & 1 & 1 \\
0 & 1 & 0 & 0 & 1 \\
1 & 1 & 1 & 1 & 0 \\
1 & 0 & \mathbf{0} & 0 & 1 \\
\hline
1 & 0 & 0 & 0 & 1
\end{array}
$$

PROBLEM 2

b1	b2	b3	b4	c1	c2	c3
1	0	1	1	1	0	0
1	1	0	1	0	1	0
0	1	1	1	0	0	1

1110111 cannot be a valid codeword given the rules in the question. Using the check matrix above we get the syndrome [1 1 1]. So we know bit number 4 was flipped. Correcting for this we get 1111111, which is a valid codeword.

PROBLEM 3

If the probability of a flip is very high, we can argue that the expected reception will be the binary complement of the original codeword, so we simply complement the received codeword and proceed as before.

PROBLEM 4

b1	b2	b3	b4	c1	c2	c3
1	0	1	0	1	0	0
0	1	0	1	0	1	0
0	1	1	0	0	0	1

(a)
Examining the check matrix, we find that every single-bit error maps into one or more columns in the matrix. Therefore this code can detect any single-bit errors.

(b)
First and fifth (single) bit errors will be detected correctly but will be ambiguous to repair since they would have the same syndrome.

The same applies to fourth and sixth (single) bit errors.

(c)
Since we can detect all 1bit errors, Hamming distance >1
Since we cannot correct all 1bit errors, Hamming distance <=2
Therefore Hamming distance = 2

Codewords: 1100101 , 1001110

PROBLEM 5

(a)
An ALOHA device may decide to send a packet at any time, possibly colliding with any of the other (n-1) devices sharing the medium.

The probability of not colliding with any of the (n-1) devices = $e^{-2\lambda L(n-1)}$

(b)
When a device using slotted ALOHA sends a packet, it may collide with any of the other (n-k-1) slotted ALOHA devices sharing the medium. The probability of that NOT happening $p_1 = e^{-\lambda L(n-k-1)}$.

In addition the packet may collide with any of the other (k) ALOHA devices. The probability of that NOT happening $p_2 = e^{-2\lambda L(k)}$.

The probability of successful transmission = $p_1 x p_2 = e^{-\lambda L(n+k-1)}$

PROBLEM 6

(a)
Using slotted ALOHA, the probability of a device NOT transmitting before time T is given by $e^{-\lambda T}$. Equating this to 1-p, we have:
$e^{-\lambda T} = 1 - p$
➔ $\lambda = -(1/T)\ln(1-p)$

(b)
For any window, every device might transmit with probability p. For a successful transmission, one device transmits and all others don't transmit within the time window. The probability of that event is therefore:
$Np(1-p)^{N-1}$

(c)
Given that one device transmitted, the probability of success is the probability that all other devices don't transmit in the same window, or: $(1-p)^{N-1}$

(d)
Expected number of successful transmissions = 1*P(successful transmission)+0*P(no transmission) + 0*P(failure to transmit due to interference). This equates to the same as in part (b): $Np(1-p)^{N-1}$

(e)
Setting the derivative of the answer in part (b) to zero, we get:
p = 1/N

PROBLEM 7

(a)
C1: 1
C2: -1
C3: 1

(b)
Example codes:

-1 -1 -1 -1 1 1 1 1
Or
1 1 1 1 -1 -1 -1 -1
Or
-1 1 -1 1 -1 1 -1 1
Or
1 -1 1 -1 1 -1 1 -1