CSEE 6861 CAD of Digital Systems
Handout: Lecture #2 (part 2)
1/28/16

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ESPRESSO Algorithm: The "EXPAND" Step

"EXPAND" Step: Overview

Basic idea/flow:

- Given current cover ***F:***

  repeat {

  - ***pick an unexpanded cube c in F***
  - ***iteratively expand into a prime implicant c****
  - ***delete any other cubes in F completely contained in c****

  } until no further cubes can be expanded (i.e. all prime)

2 Issues:

a. In what **order** to pick cubes for expansion?

b. For each picked cube, in what **direction** to expand it?

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"EXPAND" Step: Cube Order

Strategy:

- use ordering heuristic:

  (a) *preprocessing: given cover F, calculate "weight" of every literal*

  (b) using (a), *assign weight to each cube c in F*

  (c) *sort cubes by weight (lowest to highest):*

  - pick lowest weight first

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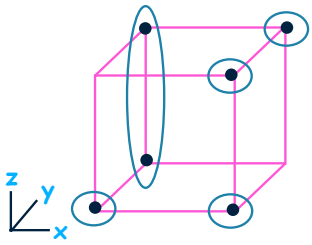
"EXPAND" Step: Cube Order

Step (a): Preprocessing -- given cover *F*, calculate "weight" of every literal

For each cube *c* in *F*:

- **if *c* has literal x:** add 1 to x entry in weight vector
- **if *c* has literal x' :** add 1 to x' entry in weight vector
- **if *c* does not have x or x' :** add 1 to x entry AND add 1 to x' entry

Example:



	x	y	z	f
x'y	0	1	-	1
x'y'z'	0	0	0	1
xyz	1	1	1	1
xy'z	1	0	1	1
xy'z'	1	0	0	1

PLA Representation

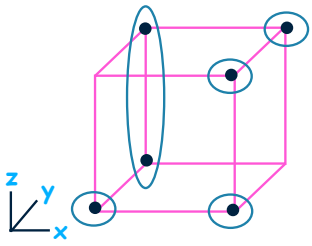
"EXPAND" Step: Cube Order

Step (a): Preprocessing -- given cover *F*, calculate "weight" of every literal

For each cube *c* in *F*:

- **if *c* has literal x:** add 1 to x entry in weight vector
- **if *c* has literal x' :** add 1 to x' entry in weight vector
- **if *c* does not have x or x' :** add 1 to x entry AND add 1 to x' entry

Example:



	x	y	z	f
x'y	0	1	-	1
x'y'z'	0	0	0	1
xyz	1	1	1	1
xy'z	1	0	1	1
xy'z'	1	0	0	1

PLA Representation

x	x'	y	y'	z	z'
3	2	2	3	3	3

Literal Weight Vector

"EXPAND" Step: Cube Order

Step (b): Calculate All Cube Weights

Example (cont.):

	x	y	z	f
x'y	0	1	-	1
x'y'z'	0	0	0	1
xyz	1	1	1	1
xy'z	1	0	1	1
xy'z'	1	0	0	1

PLA Representation

Literal Weight Vector

x	x'	y	y'	z	z'
3	2	2	3	3	3

cube	cube weight (using literal weight vector)
x' y	2 + 2 + 3 + 3 = 10
x'y'z'	2 + 3 + 3 = 8
xyz	3 + 2 + 3 = 8
xy'z	3 + 3 + 3 = 9
xy'z'	3 + 3 + 3 = 9

Cube Weight Table

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"EXPAND" Step: Cube Order

Step (b): Calculate All Cube Weights

Example (cont.):

	x	y	z	f
x'y	0	1	-	1
x'y'z'	0	0	0	1
xyz	1	1	1	1
xy'z	1	0	1	1
xy'z'	1	0	0	1

PLA Representation

Literal Weight Vector

x	x'	y	y'	z	z'
3	2	2	3	3	3

cube	cube weight (using literal weight vector)
x' y	2 + 2 + 3 + 3 = 10
x'y'z'	2 + 3 + 3 = 8
xyz	3 + 2 + 3 = 8
xy'z	3 + 3 + 3 = 9
xy'z'	3 + 3 + 3 = 9

Cube Weight Table

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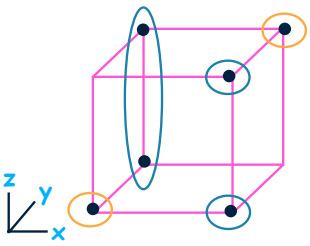
"EXPAND" Step: Cube Order

Step (c): Sort Cubes by Weight (low-to-high)

Example (cont.):



cube	cube weight (using literal weight vector)
x' y' z'	2 + 3 + 3 = 8
xyz	3 + 2 + 3 = 8
xy' z	3 + 3 + 3 = 9
xy' z'	3 + 3 + 3 = 9
x' y	2 + 2+ 3 + 3 = 10

Sorted Cube Weight Table

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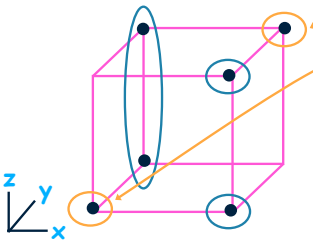
"EXPAND" Step: Cube Order

Step (c): Sort Cubes by Weight (low-to-high)

Example (cont.):



Pick lowest cost cubes to expand first:

cube	cube weight (using literal weight vector)
x' y' z'	2 + 3 + 3 = 8
xyz	3 + 2 + 3 = 8
xy' z	3 + 3 + 3 = 9
xy' z'	3 + 3 + 3 = 9
x' y	2 + 2+ 3 + 3 = 10

Intuition: these cubes are in "sparsest regions" of cover F ⇨ give them highest priority for expansion

Sorted Cube Weight Table

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"EXPAND" Step: Expansion Direction

Basic idea:

- Once ***cube c*** in ***cover F*** is selected for expansion, expand it in "good directions"

3 Heuristic Expansion Steps (in sequence): to expand a *cube c*

1. Expand ***c*** to completely contain (i.e. "swallow up") as many other cubes of ***cover F*** as possible
   - delete these completely-contained cubes immediately!
2. Once #1 done -- continue to expand ***c*** to partially overlap as many other cubes of ***cover F*** as possible
   - the goal is to "induce redundancies", which then get deleted in the IRRED step
3. Once #2 done -- continue to expand ***c*** into a "maximal size" prime implicant

Our Main Focus: Step #1

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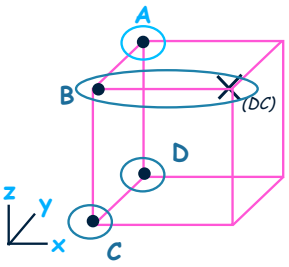
"EXPAND" Step: Expansion Direction

Example: Step #1

Suppose cube A has been picked for expansion



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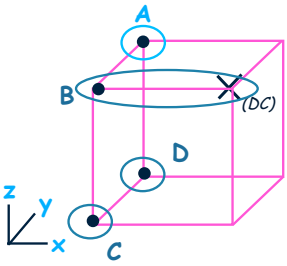
"EXPAND" Step: Expansion Direction

Example: Step #1



Suppose cube A has been picked for expansion

Define **supercube(A,X)** =
smallest cube which contains both A and X

Try all possible expansions of A (pairwise),
check if expansion hits OFF-set = "infeasible"

Candidate expansion of cube A	HITS OFF-SET?	Contains
(i) supercube(A,B) = z	**YES!**	B
(ii) supercube(A,C) = x'	NO	C, D
(iii) supercube(A,D) = x' y	NO	D

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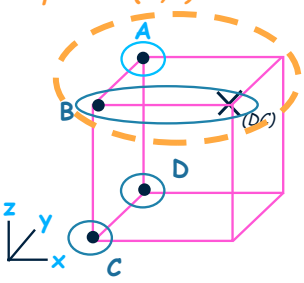
"EXPAND" Step: Expansion Direction

Example: Step #1

supercube(A,B)



"infeasible": REJECT

Suppose cube A has been picked for expansion

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"EXPAND" Step: Expansion Direction

Example: Step #1

Suppose cube A has been picked for expansion

Define **supercube(A,X)** =
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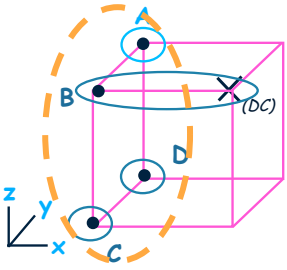
Try all possible expansions of A (pairwise),
check if expansion hits OFF-set = "infeasible"



supercube(A,C)

"feasible"

Candidate expansion of cube A	HITS OFF-SET?	Contains
(i) supercube(A,B) = z	**YES!**	B
(ii) supercube(A,C) = x'	NO	C, D
(iii) supercube(A,D) = x' y	NO	D

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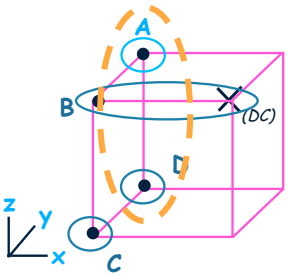
"EXPAND" Step: Expansion Direction

Example: Step #1

Suppose cube A has been picked for expansion

Define **supercube(A,X)** =
smallest cube which contains both A and X

Try all possible expansions of A (pairwise),
check if expansion hits OFF-set = "infeasible"



supercube(A,D)

"feasible"

Candidate expansion of cube A	HITS OFF-SET?	Contains
(i) supercube(A,B) = z	**YES!**	B
(ii) supercube(A,C) = x'	NO	C, D
(iii) supercube(A,D) = x' y	NO	D

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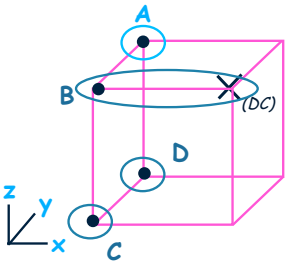
"EXPAND" Step: Expansion Direction

Example: Step #1

Suppose cube A has been picked for expansion

Define supercube(A,X) = smallest cube which contains both A and X

Try all possible expansions of A (pairwise), check if expansion hits OFF-set = "infeasible"



Candidate expansion of cube A	HITS OFF-SET?	Contains
(i) supercube(A,B) = z	**YES!**	B
(ii) supercube(A,C) = x'	NO	C, D
(iii) supercube(A,D) = x' y	NO	D

"infeasible": REJECT → (i)

"feasible" → (ii), (iii)

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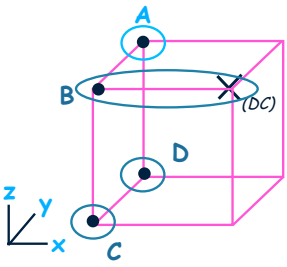
"EXPAND" Step: Expansion Direction

Example: Step #1

Suppose cube A has been picked for expansion

Pick feasible expansion of A which contains the most other cubes = supercube(A,C)



Candidate expansion of cube A	HITS OFF-SET?	Contains
(i) supercube(A,B) = z	**YES!**	B
(ii) supercube(A,C) = x'	NO	C, D
(iii) supercube(A,D) = x' y	NO	D

"infeasible" → (i)

"feasible" → (ii), (iii)

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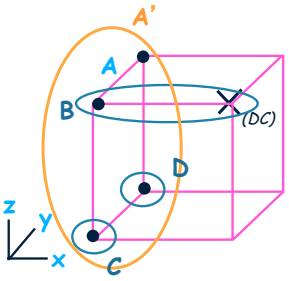
"EXPAND" Step: Expansion Direction

Example: Step #1

Suppose cube A has been picked for expansion



Pick "feasible" expansion of A which contains the most other cubes = supercube(A,C)

- expand A to A' = supercube(A,C)

	Candidate expansion of cube A	HITS OFF-SET?	Contains
"infeasible"	(i) supercube(A,B) = z	**YES!**	B
"feasible"	(ii) supercube(A,C) = x'	NO	C, D
	(iii) supercube(A,D) = x' y	NO	D

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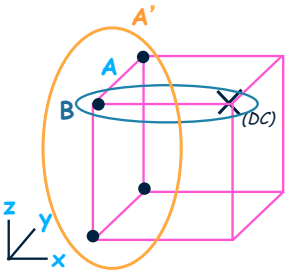
"EXPAND" Step: Expansion Direction

Example: Step #1

Suppose cube A has been picked for expansion



Pick "feasible" expansion of A which contains the most other cubes = supercube(A,C)

- expand A to supercube(A,C)
- delete C, D

	Candidate expansion of cube A	HITS OFF-SET?	Contains
"infeasible"	(i) supercube(A,B) = z	**YES!**	B
"feasible"	(ii) supercube(A,C) = x'	NO	C, D
	(iii) supercube(A,D) = x' y	NO	D

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