More ESPRESSO Examples
Example #4: Multi-Output Minimization

Illustrates EXPAND/IRRED (multi-output)

Initial cover ("seed")

input part | output part
---|---
x | y | z | f1 | f2
0 | - | 1 | 1 | 0
1 | - | 0 | 1 | 0
0 | 0 | - | 0 | 1
- | 0 | 0 | 0 | 1
- | 1 | 1 | 0 | 1

PLA Representation = "cubical complex"

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Example #4: Multi-Output Minimization

input part | output part
---|---
x | y | z | f1 | f2
0 | - | 1 | 1 | 0
1 | - | 0 | 1 | 0
0 | 0 | - | 0 | 1
- | 0 | 0 | 0 | 1
- | 1 | 1 | 0 | 1

PLA Representation = "cubical complex"

corresponding 2-level implementation
Example #4: Multi-Output Minimization

**PLA Representation**

= l

input part | output part
---|---
0 1 0 | f1 f2
1 0 1 | 0 1
0 0 0 | 0 1
- 0 0 1
- 1 1 0

**PLA Representation**

= "cubical complex"

**Expand step**

(multi-output)

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**NEW!**

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**NEW!**

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Example #4: Multi-Output Minimization

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
<th>z</th>
<th>f1</th>
<th>f2</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-1</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>-1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>-1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

Cube now redundant

AND gate now redundant

DELETE

DELETE

IRREDUNDANT step (multi-output)
Example #4: Multi-Output Minimization

<table>
<thead>
<tr>
<th>input part</th>
<th>output part</th>
</tr>
</thead>
<tbody>
<tr>
<td>x y z</td>
<td>f1 f2</td>
</tr>
<tr>
<td>0 1 0</td>
<td>1 1</td>
</tr>
<tr>
<td>1 0 1</td>
<td>0 0</td>
</tr>
<tr>
<td>0 0 0</td>
<td>0 1</td>
</tr>
<tr>
<td>1 1 0</td>
<td>1 0</td>
</tr>
</tbody>
</table>

PLA Representation = "cubical complex"

IRREDUNDANT step (multi-output)

FINAL MINIMIZED COVER (multi-output)
Introduction to ESPRESSO: Examples

Example #5: Multi-Output Minimization

Illustrates complete iteration loop (multi-output)

Initial cover ("seed")

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
<th>z</th>
<th>f1</th>
<th>f2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>-</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>-</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>-</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>-</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

PLA Representation = "cubical complex"

corresponding 2-level implementation
Example #5: Multi-Output Minimization

PLA Representation

Expand step... (multi-output)

No change: all primes
Example #5: Multi-Output Minimization

PLA Representation
= cubical complex

IRRED step... (multi-output)
No change

REDUCE step... (multi-output)
cube order: A, B
Example #5: Multi-Output Minimization

PLA Representation = cubical complex

REDUCE step...: BEFORE (multi-output)

cube order: A, B

REDUCE step...: AFTER (multi-output)

cube order: A, B
Example #5: Multi-Output Minimization

PLA Representation

- Cubical complex
- Input part
- Output part

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th>f1</th>
<th>f2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-</td>
<td>1</td>
<td>1</td>
<td>0</td>
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<tr>
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<tr>
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<td>-</td>
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<td>1</td>
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<tr>
<td>-</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

PLA Representation = “cubical complex”

EXPAND step (2nd time!) … (multi-output)

cube order: A’, B

Example #5: Multi-Output Minimization

PLA Representation

- Cubical complex
- Input part
- Output part

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th>f1</th>
<th>f2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-</td>
<td>1</td>
<td>1</td>
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<td>1</td>
</tr>
</tbody>
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PLA Representation = “cubical complex”

EXPAND step (2nd time!) … (multi-output)

cube order: A’, B

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Introduction to ESPRESSO: Examples

Example #5: Multi-Output Minimization

PLA Representation = "cubical complex"

EXPAND step (2nd time!) … (multi-output)

cube order: A', B

Introduction to ESPRESSO: Examples

Example #5: Multi-Output Minimization

PLA Representation = "cubical complex"

EXPAND step (2nd time!) … (multi-output)

cube order: A', B
Introduction to ESPRESSO: Examples

Example #5: Multi-Output Minimization

PLA Representation = cubical complex

input part  output part
\[
\begin{array}{ccc|cc}
  x & y & z & f1 & f2 \\
  1 & 1 & 1 & 1 & 0 \\
  0 & 1 & 1 & 1 & 1 \\
  -1 & 0 & 0 & 1 & 0 \\
  0 & 1 & - & 0 & 1 \\
  - & 0 & 1 & - & 0 & 1 \\
\end{array}
\]

IRRED step (2nd time!) ...
(multi-output)

C redundant: DELETE
Example #5: Multi-Output Minimization

**PLA Representation**

- **Input Part:**
  - x
  - y
  - z

- **Output Part:**
  - f1
  - f2

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
<th>z</th>
<th>f1</th>
<th>f2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
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<tr>
<td>-</td>
<td>1</td>
<td>0</td>
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<td>1</td>
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<td>1</td>
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</table>

PLA Representation = "cubical complex"

**IRRED step:**

- (multi-output)

"C" redundant: DELETE

**Final Minimized Cover:**

(multi-output)