PRIME GENERATION PROBLEM: Generate all Primes

Hypercube for a function \( f \):

\[ f = x'y + x'y'z + xz + xz' \]

Step 1: Termination rules B1-B4 + U1 do not apply. Goto step 2. (See PLA below.)

Step 2: Pick splitting variable. A good heuristic is described in Handbook [21]. Here, \( x \) is binary (appears as both \( x + x' \)). It also contributes to the most implicants. Therefore, pick \( x \).

Recursively compute:
(i) \( A_1 = x \cdot \text{Primes}(F_x) \)
(ii) \( A_0 = x' \cdot \text{Primes}(F_{x'}) \)

First, compute \( F_x + F_{x'} \) (cortors):

PLA:

<table>
<thead>
<tr>
<th>( F_x )</th>
<th>( F_{x'} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0 0 1 1</td>
</tr>
<tr>
<td>2</td>
<td>0 1 1 1</td>
</tr>
<tr>
<td>3</td>
<td>1 1 1 1</td>
</tr>
<tr>
<td>4</td>
<td>0 1 1 1</td>
</tr>
<tr>
<td>5</td>
<td>1 1 1 1</td>
</tr>
</tbody>
</table>

Compute \( F_x \):

\[ F_x = \begin{array}{cc}
0 & 1 \\
0 & 1 \\
0 & 1 \\
0 & 1 \\
0 & 1 \\
\end{array} \]

(Cont.)
Step 2. (cont.)

Compute \( F_x \):

\[
\begin{array}{c|ccc|c}
  x & y & z & f \\
  \hline
  0 & 0 & 0 & 1 \\
  1 & 0 & 0 & 0 \\
  1 & 1 & 0 & 0 \\
  1 & 1 & 1 & 0 \\
  1 & 1 & 1 & 0
\end{array}
\]

\[ F = \begin{bmatrix}
  1 \\
  2 \\
  3 \\
  4 \\
  5
\end{bmatrix}
\Rightarrow \begin{bmatrix}
  y \\
  z \\
  f
\end{bmatrix}
\]

Next, compute \( \text{Primes}(F_x) \) and \( \text{Primes}(F_x') \).

\( \Rightarrow \) Both \( F_x \) and \( F_x' \) are terminal cases.

\( F_x \): \boxed{\text{Rule B2}} - single input dependence.

\( F_x \) is a tautology. Return the universal cube:

\[ \text{Primes}(F_x) = \exists x \top \]

\( F_x' \): \boxed{\text{Rule U1}} - unate function.

\( F_x' \) is unate. Do "SCC", then return the result:

\[ \text{Primes}(F_x') = \text{SCC}(F_x') = \exists y, z \top \]

\( \text{SCC} \{ y \top, y \top \} = \exists y, z \top \)
**PRIME GENERATION PROBLEM**

*Example*

**Step 2. (cont.)**

Finally, compute $A_1 + A_0$:

where

$$A_1 = x \cdot \text{Primes}(F_x)$$

$$= x \times \{ y \} = \{ x'y \}$$

and

$$A_0 = x' \cdot \text{Primes}(F_{x'})$$

$$= x' \times \{ y, z \} = \{ x'y, x'z \}$$

**Compute consensus** $(A_1, A_0)$:

- take pairwise consensus, of each cube in $A_1$ with each cube in $A_0$:

$$\text{consensus}(x, x'y) = y$$

$$\text{consensus}(x, x'z) = z$$

$$\Rightarrow \text{consensus}(A_1, A_0) = \{ xy, xz \}$$

**Take union of the 3 sets:**

$A_1 \cup A_0 \cup \text{consensus}(A_1, A_0) = \{ x, x'y, x'z, y, z \}$

**Apply SCC operator:**

$$\text{SCC}(\{ x, x'y, x'z, y, z \}) = \{ x'y, xz \}$$

**Return result:** $\{ x'y, xz \} = \text{set of all primes of } f$.
Final Recursive Flow:

At root node: combining recursive results from 2 child nodes:

Primes(f) = SCC(A1 U AO U consensus (A1/AD)),

where:

A1 = x, Primes(Fx) = x \cdot \{0, \bar{x}\} = \{x, \bar{x}\},

A0 = \bar{x}, Primes(Fx) = \bar{x} \cdot \{y, z\} = \{\bar{x}y, \bar{x}z\},

consensus (A1, A0) = \{y, z\} (see page 3),

A1 U AO U consensus (A1, A0) = \{x, y, z\, \bar{x}\}

and SCC(A1 U AO U consensus (A1, A0)) = \{x, y, z\}

(where cubes x'y + x'z have been deleted)