CSEE E6861y **Homework 2** Handout 13
Prof. Steven Nowick February 11, 2016

This homework is due at the *beginning of class* on Thursday, February 18.

NOTE: A correct answer without adequate explanation or derivation will have points deducted. To get full credit, (a) write legibly/type, and (b) show all work (label relevant items, show deriviations, include explanations).

1. (15 points) **Cofactors: Single-Output (part I).** For this problem, you will use the example cover in H/S problem #10 in chapter 5.8 (see Handout #14), and perform different cofactor operations on this cover. (Note: *do not do H/S problem #10*, just use its PLA representation to solve the following cofactor problems.) Follow the approach shown in Handout #12, and show all work.
   (a) Cofactor the cover with respect to literal x'.
   (b) Cofactor the cover with respect to literal y.
   (c) Cofactor the cover with respect to cube w'y.

2. (15 points) **Cofactors: Multi-Output (part II).** Do H/S problem #5 in chapter 5.8 (see Handout #14). Clearly show all of your work. Refer also to solved multi-output cofactor problems in Hachtel/Somenzi ch. 5.8 (in Handout #14), such as problem #4, as well as to any relevant discussion in Hachtel/Somenzi within Handout #10. You will learn how to generalize the notion of cofactor from single-output functions to multi-output functions.

3. (10 points) **Unateness of Covers and Functions.**
   (a) Refer to the cover in H/S problem #13 in ch. 5.8 (see Handout #14). In what variables is this cover unate? (Note: *do not do H/S problem #13*, just use its PLA representation to answer the question.) Show all work.
   (b) Do H/S problem #9 in ch. 5.8 (see Handout #14). Show all work.

4. (20 points) **Fast Tautology Check.** Given the cover F below, determine if the corresponding function f is a tautology. Follow the method of the Tautology Handout (Handout #11), and as illustrated in the example on Handout #12. Only use rules B1-B4, and U1 and U2. (Ignore rules M1 and M2 for now.)

$w\ x\ y\ z$	f
- 0 1 -	1
1 1 0 0	1
1 - - -	1
0 1 - -	1
0 - - 1	1

   For choice of splitting variable, use the following guideline: (i) your primary choice is to pick a unate splitting variable, to exploit Rule U2 whenever possible; (ii) if there are none, your secondary choice is to follow the "Choice of Splitting Variable" guidelines in Handout #11. *Show your work:* clearly draw the resulting recursion tree, indicating splitting all variables, and showing the results derived and returned *at each step*.

5. (20 points) **Essentials #1: given the set of all primes.** You are given a single-output binary-valued function f, given by the following cover F:

$v\ w\ x\ y\ z$	f
1 1 0 1 -	1
0 1 - - 0	1
1 - 0 - 0	1
1 1 - 1 0	1
1 - 0 1 0	1

Assume this cover is the ON-set of the function, and that the DC-set is empty. You are to determine if the cube, $vwyz'$, listed as the second-to-last row in this PLA file, is *essential.*

Follow the "containment" approach outlined in Sections 4 and 5, and the later complete example, in Handout #12. Once you formulate the problem as a tautology-check problem, you do *not* need to work through the recursion tree. Simply present the formulation, and give the final answer by inspection with a clear justification.

6. (20 points) **Finding a Maximal Size Prime.**

In this exercise, you will solve an important CAD problem: given an implicant p (which is not yet prime) and OFF-set R, how to expand p into the *largest prime implicant* which contains it. Here, 'largest' means the prime implicant with fewest literals, i.e. which contains the most minterms.

Your solution will be a systematic method to solve "Step #3" of espresso's expand operation (after trying to cover other cubes, and overlap other cubes), as illustrated in class and in the Lecture #3 slides.

Introduction: Given an implicant p which is not prime, there may be several different directions to expand p into a prime. The goal is to expand it into *largest possible prime* which contains p, called a *maximal prime*. In this problem, you will show how the problem of finding a maximal prime containing p can be formulated as a *covering problem:* the same technique we used in Quine-McCluskey Method where we formed a covering table (i.e. prime implicant table).

The discussion below guides you through some intuition and hints, and then you will complete the final formulation and solution of this problem on your own.

Assumptions: Assume you are given an implicant $p = ab'cde$ to expand, and
OFF-set $R = \{a'bcd', bc'e, a'e', ad'e'\}$.

Observations: Product p is an implicant, so it does not intersect any OFF-set cube. This is clear, since p has *distance-1 or greater* from each OFF-set cube. Product p is expanded by **"raising variables"**; that is, by changing a literal or its complement into a don't-care. For example, p can be expanded by "raising" its literal a into a don't-care; the result is the new product $b'cde$. As variables are raised, the distance between the product and OFF-set cubes may decrease. The goal is to raise variables of p, but *still* insure that the expanded product does not intersect any OFF-set cube. That is, the goal is to insure that the expansion is *legal.*

You are to formalize this expansion problem below.

(a) *Write a constraint equation* (similar to Petrick's method), in product-of-sums form, which describes *all legal expansions* of product p. (Do not solve it.) This should be a *unate expression* (i.e. as in the product-of-sums expression used in Petrick's method, which does not have any propositions complemented).

Hint: For each cube in the OFF-set, generate one clause in your product-of-sums equation. The clause describes conditions to insure that p will *not* intersect that particular OFF-set cube. Use proposition A to be true (i.e. 1) to mean that variable a is **not raised**, B means that variable b is not raised, etc. The result will be a constraint equation, with 4 clauses, which constrains the expansion of implicant p, and insures that p will never be expanded to intersect any OFF-set cube.

(b) *Briefly and clearly explain* how this constraint equation can be used to find an expansion of p into a *maximal prime implicant* which contains it.