# An Introduction to Hazard-Free Logic Synthesis (Fundamental Mode) 

Steven M. Nowick<br>Columbia University<br>(nowick@cs.columbia.edu)

July 15, 2002

## Goal

Given: a Boolean function
Design: a hazard-free circuit implementation


## Goal

Given: a Boolean function
Design: a hazard-free circuit implementation


## Outline

Basics: Hazards
Part I. 2-Level Logic
Part II. Multi-Level Logic and
Technology Mapping
Conclusions

## Basics

1) "Input Transition"
= "multiple-input transition", "multiple-input change" [MIC]
= a change from one input vector to another


Assume: "clean" input transitions => no glitches!

## Basics

2) Circuit Model

Assume an "unbounded wire delay" model
... gates and wires may have arbitrary (finite) delays!


## The Goal

Given a specified input transition, synthesize a circuit impltn. with no "combinational hazards" for this transition (i.e. no possible glitch on outputs!)

assuming above circuit \& environmental models....

## The Goal

Given a specified input transition, synthesize a circuit impltn. with no "combinational hazard" for this transition (i.e. no possible glitch on outputs!)

assuming above circuit \& environmental models.

## Basics

3) Environmental Model

Assume "generalized fundamental mode"
... after an input transition, no new inputs may arrive until the circuit has stabilized!


## Key Differences from "QDI" Hazard-Free Design

1. Combinational Circuit Model: now more robust!

- circuits correct for arbitrary gate + wire delays
- ... vs. QDI: uses "isochronic fork" assumption

2. Environmental Model: "generalized fundamental mode

- now, timing assumptions on environment (1-sided
- ... vs. QDI: "input/output mode" (= none)


## Basics: Combinational Hazards

Two types of combinational hazards:

1. Function Hazard:

- inherent in combinational function

2. Logic Hazard:

- inherent in circuit implementation



## Function Hazards



## Function Hazards: Summary

Function hazards: cannot be removed

- inherent in function itself
- cannot quarantee glitch-free logic implementation [Unger]

Therefore, only consider function hazard-free transitions:

- most "specified behaviors" = naturally monotonic (not glitchy)

Sequential synthesis methods:

- must not introduce function hazards

Burst-mode: uses ...

- constrained 'state minimization' + 'state assignment' steps
- always succeeds: no undesired function hazards introduced...


## Logic Hazards

Now, assume function hazard-free input transitions....
Logic Hazard = property of a given circuit implementation

Def. Logic Hazard: Given combinational function f, circuit implementation C , and an input transition t .

If $f$ is function hazard-free for input transition $t$, but implementation C may glitch during transition t , then circuit C has a logic hazard for transition t .
Otherwise, circuit C is logic hazard-free for transition t.


## Logic Hazards



A Different Implementation

"input change": $D: 1-->0$
$(A B C=011)$

## Part I

Two-Level Logic

## Part I: Outline

- Problem \#1: Eliminating Logic Hazards for One Input Transition
- Problem \#2: Eliminating Logic Hazards for Several Input Transitions
- 2-Level Hazard-Free Logic Minimization:
a Complete Example
- Existence of a Hazard-Free Solution
- An Alternative Approach: Using GC-Elements

PROBLEM \#1: Eliminating Logic Hazards for One Input Transition

Given: a combinational function f , and a function hazard-free input transition t .

Goal: find a 2-level (AND-OR) implementation of $f$ which is logic hazard-free for input transition $t$.

## SUMMARY:

## Eliminating Logic Hazards for One Input Transition





Eliminating Hazards: 1->1 Transition
AB



Eliminating Hazards: 1->1 Transition
AB




Eliminating Hazards: 1->1 Transition


Eliminating 1->1 Hazard: Summary


## Eliminating Hazards:

"Dynamic Transition" (1->0 or 0->1)

| AB |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| CD | 00 | 01 | 11 | 10 |
|  | 0 | 0 | $\square$ | 0 |
| 00 |  |  |  |  |
|  | - |  | $D$ |  |
| 11 |  |  | 1 | 0 |
| 10 | O | (1) | U | 0 |

"input change": AC: 00->11 ( $\mathrm{BD}=11$ )


Example Circuit

## Eliminating Hazards: 1->0 Transition




Problem \#1: 1-to-1 "partial transition" is hazardous:

- violates 1->1 covering requirement

Eliminating Hazards: 1->0 Transition


Problem \#1: "required cube" for partial transition - ... not covered by any product!

Eliminating Hazards: 1->0 Transition



Requirement \#1
Solution: cover the "required cube" for each partial transition - ... by some product

Eliminating Hazards: 1->0 Transition


Problem \#2: entire dynamic 1-to-0 transition still hazardous!

Eliminating Hazards: 1->0 Transition



Eliminating Hazards: 1->0 Transition


Eliminating Hazards: 1->0 Transition

glitch!


Eliminating Hazards: 1->0 Transition



Eliminating 1->0 Hazard: Summary

| 0 | 0 | 0 | 0 |  |
| :--- | :---: | :---: | :---: | :---: |
| 0 | 1 | 1 | 0 |  |
| 0 | 0 | 0 | 0 |  |
| 0 | 1 | 0 | 0 |  |
| "inliegal <br> intersection" |  |  |  |  |



Eliminating 1->0 Hazard: Summary


## Eliminating 1->0 Hazard: "Privileged Cubes"



## Eliminating 1->0 Hazard: Summary

hazardous

| 0 | 0 | 0 | 0 |
| :--- | :---: | :---: | :---: |
| 0 | 1 | 1 | 0 |
| 0 |  | 0 | 0 |
| 0 | 1 | 0 | 0 |

illegal intersection
hazard-free

| 0 | 0 | 0 | 0 |
| :--- | :--- | :--- | :--- |
| 0 | 1 | 1 | 0 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 0 | 0 |

legal intersection
"privileged cube": must not be illegally intersected by any product


## PROBLEM \#2: Eliminating Logic Hazards for Several Input Transitions

## "2-Level Hazard-Free Logic Minimization Problem"

## Given:

- a Boolean function
- a specified set of input transitions


## Find;

- a minimum-cost 2-level implementation which is hazard-free for each specified input transition (i.e, guaranteed not to glitch)

Goals and Assumptions:

- produce hazard-free combinational circuit:
- guaranteed glitch-free, regardless of gate+wire delays
- inputs: assumed to be glitch-free


## 2-Level Hazard-Free Logic Minimization Problem

## Equivalent Goal

Find a 2-level circuit implementation, where:

- no privileged cube is "illegally intersected" by a product; and
- each required cube is completely contained in some product.


## "Dynamic Hazard-Free (DHF) Prime Implicants"

 has illegal intersection

DHF-Prime Implicant = | a maximal implicant which |
| :---: |
| has no "illegal intersections" |
| with any privileged cubes |



DHF-Prime Implicant: no illegal intersections

## 2-Level Hazard-Free Logic Minimization Problem (cont.)

Revised Goal (version \#2):

Find a 2-level circuit implementation:

- ... using only DHF-prime implicants,
- ... where each required cube is completely covered by some product.

2-Level Logic Minimization: a Comparison (Classic vs. Hazard-Free)

In each case, solve a "covering problem":
<"objects to be covered", "covering objects">

- Classic (Quine-McCluskey method, espresso-exact, ...): <on-set minterms, prime implicants>
- Hazard-Free (Nowick/Dill [92]):
<required cubes, DHF-prime implicants>


## 2-Level Logic Minimization: a Comparison

## Classic Method: Non-Hazard-Free

Step 1: Generate All Prime Implicants
Step 2: Generate Prime Implicant Table
Step 3: Solve Covering Problem

## New Method: Hazard-Free [Nowick/Dill '92]

Step 1: Generate All DHF-Prime Implicants
Step 2: Generate DHF-Prime Implicant Table
Step 3: Solve Covering Problem

## 2-Level Hazard-Free Logic Minimization: a Complete Example


[from: Nowick/Dill, ICCAD' 92; IEEE Trans. On CAD Aug.' 95]

Boolean Function +
4 (function hazard-free) input transitions

## 2-Level Hazard-Free Logic Minimization: a Complete Example



## Required Cubes:

Each required cube must be completely contained in some product


## 2-Level Hazard-Free Logic Minimization: a Complete Example

Step 1: Generate All DHF-Prime Implicants


## 2-Level Hazard-Free Logic Minimization: a Complete Example

Step 1: Generate All DHF-Prime Implicants


Approach: 2 steps
$\Rightarrow$ Generate All Prime Implicants

- Reduce to DHF-Prime Implicants

Total: 7 Prime Implicants

## 2-Level Hazard-Free Logic Minimization: a Complete Example

Step 1: Generate All DHF-Prime Implicants


Approach: 2 steps

- Generate All Prime Implicants $\Rightarrow$ Reduce to DHF-Prime Implicants \% = privileged cube


## 2-Level Hazard-Free Logic Minimization: a Complete Example

Step 1: Generate All DHF-Prime Implicants


## 2-Level Hazard-Free Logic Minimization: a Complete Example

Step 1: Generate All DHF-Prime Implicants
Approach: 2 steps

- Generate All Prime Implicants

$\Rightarrow$ Reduce to DHF-Prime Implicants

Some primes have no illegal intersections
$=>$ they are DHF-primes

## 2-Level Hazard-Free Logic Minimization: a Complete Example

Step 1: Generate All DHF-Prime Implicants


Approach: 2 steps

- Generate All Prime Implicants
$\Rightarrow$ Reduce to DHF-Prime Implicants

Other primes have
illegal intersections
=> they must be reduced

## 2-Level Hazard-Free Logic Minimization: a Complete Example

Step 1: Generate All DHF-Prime Implicants First reduction of prime implicant


## 2-Level Hazard-Free Logic Minimization: a Complete Example

Step 1: Generate All DHF-Prime Implicants
Second reduction of prime implicant $\frac{\text { DISCARD: contained }}{\text { in a DHF-Prime }}$


## 2-Level Hazard-Free Logic Minimization: a Complete Example

Step 1: Generate All DHF-Prime Implicants


## 2-Level Hazard-Free Logic Minimization: a Complete Example

Step 1: Generate All DHF-Prime Implicants


## 2-Level Hazard-Free Logic Minimization: a Complete Example

Step 1: Generate All DHF-Prime Implicants


Final Result: 6 DHF-Prime Implicants

## 2-Level Hazard-Free Logic Minimization: a Complete Example

## Step 2: Generate DHF-Prime Implicant Table

|  |  |  | $\times$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\times$ |  |  |  |  |  |
|  | x | x |  |  |  |
|  |  |  |  | x |  |
|  |  |  |  |  | x |



2-Level Hazard-Free Logic Minimization: a Complete Example

Final Hazard-Free Circuit (minimum-cost):


2-Level Hazard-Free Logic Minimization: a Complete Example

Final Non-Hazard-Free Circuit (minimum-cost):


## 2-Level Hazard-Free Logic Minimization: Another Example



Non-hazard-free: min cost = 3 products


Hazard-free: min cost $=3$ products

## Existence of a Hazard-Free Solution

## Challenge: a hazard-free 2-level implementation does not always exist!



New Example: add 1 more input transition

## Existence of a Hazard-Free Solution



Existence of a Hazard-Free Solution

| $\begin{array}{r} \text { DHF-Prim } \\ \text { Implicant Te } \end{array}$ | able | DHF--Prime Implicants |  |  |  | P6 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | P1 |  |  |  |  |  |  |
| ac' |  |  |  | X |  |  |  |
| $a^{\prime} c^{\prime} d^{\prime}$ | x |  |  |  |  |  |  |
| 'bc' |  | X | X |  |  |  |  |
|  |  |  |  |  | x |  |  |
| bcd |  |  |  |  |  | X |  |
| $\triangle \mathrm{new}$ ] ${ }_{\text {abd }}$ |  |  |  |  |  |  |  |

## Existence of a Hazard-Free Solution

Conclusion:

- asynchronous sequential synthesis methods:
must produce functions for which hazard-free implementations exist!

Burst-Mode Synthesis Methods: impose constraints on

- state minimization
- state assignment
... to generate Boolean functions where all logic hazards can always be eliminated

Always guarantee a hazard-free solution exists

## An Alternative Approach: using "Generalized C-Elements"

## Alternative to 2-Level Logic

Target = "Generalized C-element (GC)":


## Hazard-Free Logic Using GC-Elements



Function


## A Reading List

## Hazard Basics:

- S. Unger, Asynchronous Sequential Switching Circuits, Wiley Interscience, 1969
- J. Beister, "A Unified Approach to Combinational Hazards", IEEE Transactions on Computers, vol C-23, no. 6, 1974
- S.M. Nowick, Automatic Synthesis of Burst-Mode Asynchronous Controllers, PhD Thesis, Stanford University, March 1993 (revised technical report, Stanford Computer Systems Lab CSL-TR-95-686, Dec. 1995).
- S.M. Nowick and D.L. Dill, "Exact Two-Level Minimization of Hazard-Free Logic with Multiple-Input Changes", IEEE Transactions on Computer-Aided Design, vol. 14, pp. 986-997, August 1995


## Two-Level Hazard-Free Logic Minimization:

Basic Method: (first complete solution) exact hazard-free minimization

- S.M. Nowick and D.L. Dill, "Exact Two-Level Minimization of Hazard-Free Logic with Multiple-Input Changes", IEEE Transactions on Computer-Aided Design, vol. 14, pp. 986-997, August 1995


## A Reading List (cont.)

## Two-Level Hazard-Free Logic Minimization (cont.):

HFMIN: binary \& symbolic (exact) hazard-free minimization

- R.M. Fuhrer and S.M. Nowick, Sequential Optimization of Asynchronous and Synchronous Finite-State Machines: Algorithms and Tools. Kluwer Academic, 2001.
Recent Methods: Exact Solutions
- "IMPYMIN": M. Theobald and S.M. Nowick, "Fast Heuristic and Exact Algorithms for Two-Level Hazard-Free Logic Minimization", IEEE Transactions on Computer-Aided Design, vol. 17, pp. 1130-1147, November 1998
- C. Myers and H. Jacobson, "Efficient Exact Two-Level Hazard-Free Logic Minimization", Async-01 Symposium (IEEE Int. Symp. On Advanced Rsrch. In Asynchronous Circuits and Systems), pp. 64-73, March 2001
- J. Rutten, M. Berkelaar, et al., "An Efficient Divide and Conquer Algorithm for Exact Hazard-Free Logic Minimization", Design, Automation and Test in Europe Conference (DATE), pp. 749-754, February 1998.
Recent Methods: Heuristic Solutions
- "ESPRESSO-HF": M. Theobald and S.M. Nowick, "Fast Heuristic and Exact Algorithms for Two-Level Hazard-Free Logic Minimization", IEEE Transactions on Computer-Aided Design, vol. 17, pp. 1130-1147, November 1998


## Part II

> Multi-Level Logic and Technology Mapping

## Goal: Hazard-Free Multi-Level Logic

## Strategy

Start with: hazard-free 2-level logic
Apply: hazard-non-increasing multi-level transformations


## Hazard-Non-Increasing Multi-Level Transforms

A Large Menu of "Safe Transforms": [Unger, Kung]

- Associative Law
- Factoring
- DeMorgan' s Law
- ... Many others:
- Kernel \& Cube Factoring
- Dual Global Flow
- Double Inversion
- Tree Decomposition of a Gate



## Associative Law (2)

Example: decomposing large fan-in gates


This transform may introduce hazards in 'speed-independent' or 'QDI'
circuits.


## DeMorgan’s Law

## Example:

$$
\begin{aligned}
& (A+B+C+D)^{\prime}=A^{\prime} * B^{\prime} * C^{\prime} * D^{\prime} \\
& \left(A * B^{*} C^{*} D\right)^{\prime}=A^{\prime}+B^{\prime}+C^{\prime}+D
\end{aligned}
$$

Example: 2-Level Circuit (cont.)


## Summary

Hazard-Non-Increasing Transforms:

- Allow hazard-free decomposition into simple gates (always!)
- Wide \& flexible range of safe transforms:
- much overlap with 'scripts' of Synopsys Design Compiler

■ Less restrictive than $\underline{Q D I}$ or speed-independent transforms:

- many safe "fundamental mode" multi-level transforms fail with QDI [e.g. associative law]


## Hazard-Free Technology Mapping

1. Basic approach:

- Siegel, De Micheli [DAC’ 93]

2. For improved "average-case performance":

- basic: Beerel et al. [Async' 96]
- transistor-level optimization: James, Yun [Async' 98]

3. For complex CMOS gates:

- Kudva et al. [DAC’ 96]


## A Reading List

## Hazard-Free Multi-Level Logic:

- S. Unger, Asynchronous Sequential Switching Circuits, Wiley Interscience, 1969
- D. Kung, "Hazard-non-increasing gate-level optimization algorithms", IEEE International Conference on CAD (ICCAD), pp. 631-634, Nov. 1992
- B. Lin and S. Devadas, "Synthesis of Hazard-Free Multi-Level Logic Under Multiple-Input Changes from Binary Decision Diagrams", IEEE Transactions on Computer-Aided Design, vol. 14:8, pp. 974-985, August 1995


## Hazard-Free Technology Mapping:

- P. Siegel, G. De Micheli, and D. Dill, "Automatic Technology Mapping for Generalized Fundamental Mode Asynchronous Designs," IEEE Design Automation Conference (DAC), pp. 61-67, June 1993
- P.A. Beerel, K.Y. Yun, and W.C. Chou, "Optimizing Average-Case Delay in Technology Mapping of Burst-Mode Circuits", Async Symposium (IEEE Intl. Symposium on Advanced Research in Async. Circuits and Systems), PP. 244-259, March 1996.


## A Reading List (cont.)

## Hazard-Free Technology Mapping (cont.):

- K. James and K.Y. Yun, "Average-case optimized transistor-level technology mapping of extended burst-mode circuits", Async Symposium (IEEE Intl. Symposium on Advanced Research in Async. Circuits and Systems), PP. 70-79, April 1998.
- P. Kudva, G. Gopalakrishnan, H. Jacobson, and S. Nowick, "Synthesis of Hazard-Free Customized CMOS Complex-Gate Networks Under Multiple-Input Changes", IEEE Design Automation Conference (DAC), pp. 77-82, June 1996

