

# **CSEE 4823 Advanced Logic Design**

## **Handout: Lecture #6**

**9/22/16**

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Prof. Steven M. Nowick  
*nowick@cs.columbia.edu*

Department of Computer Science (and Elect. Eng.)  
Columbia University  
New York, NY, USA

## **Iterative Circuits:**

### **Example #2**

***(Mealy-machine based)***

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## Pattern Detector: Verbal Description

**General Problem Statement:** given a string X of input bits, which are expected to follow the repeated pattern "0011" (i.e. 001100110011...), produce a corresponding string Z of output bits which flag any error locations with a '1' bit (and all correct locations with a '0' bit).

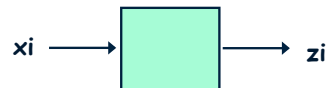
**More Detailed Requirements:** flagging errors

- the machine first expects a '0' input (if it receives any '1' inputs, stay in initial state and flag these erroneous inputs)
  - after it receives its first '0' bit, it outputs a '0' (no error), and waits for a second bit
    - if second bit is '0', no error (output '0')
    - if second bit is '1', has error (output '1')
  - ... in each case, absorb the second input, and prepare for the next pair of '11' bits
  - if the machine receives any more '0' inputs, stay in the current state and flag these erroneous inputs
  - after it receives its first '1' bit, it outputs a '0' (no error), and waits for a second bit
    - if second bit is '1', no error (output '0')
    - if second bit is '0', has error (output '1')
  - ... in each case, absorb the second input, and prepare for the next pair of '00' bits
- ... go to top and repeat

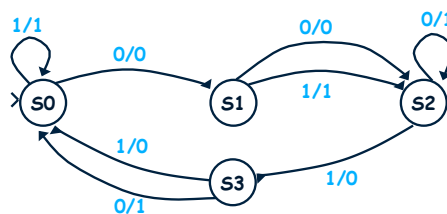
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## Iterative Circuit Synthesis: Mealy-based

**FSM Block Diagram:** *bit-serial processing*



**Mealy State Diagram:** *bit-serial processing*



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## Iterative Circuit Synthesis: Mealy-based

Symbolic State Table:

PS	input $x_i$	NS	output $z_i$
S0	0	S1	0
S0	1	S0	1
S1	0	S2	0
S1	1	S2	1
S2	0	S2	1
S2	1	S3	0
S3	0	S0	1
S3	1	S0	0

State Assignment (i.e. State Encoding):

State	Assigned Code $a_i b_i$
S0	0 0
S1	0 1
S2	1 1
S3	1 0

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## Iterative Circuit Synthesis: Mealy-based

Encoded State Table:

PS $a_i b_i$	input $x_i$	NS $a_{i+1} b_{i+1}$	output $z_i$
00	0	01	0
00	1	00	1
01	0	11	0
01	1	11	1
11	0	11	1
11	1	10	0
10	0	00	1
10	1	00	0

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## Iterative Circuit Synthesis: Mealy-based

Karnaugh Maps:

		xi	
		0	1
ai bi	00	0	0
	01	1	1
	11	1	1
	10	0	0

		xi	
		0	1
ai bi	00	1	0
	01	1	1
	11	1	0
	10	0	0

		xi	
		0	1
ai bi	00	0	1
	01	0	1
	11	1	0
	10	1	0

Final 2-Level (Sum-of-Products) Minimized Equations -- with some logic optimization added

$$ai+1 = bi$$

$$bi+1 = ai' xi' + xi' bi + ai' bi$$

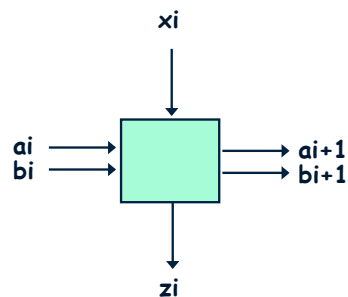
$$zi = xi ai' + xi' ai = xi \oplus ai$$

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## Iterative Circuit Synthesis: Mealy-based

Iterative Cell: Gate-Level Implementation

regular instance (i.e. typical cell): use equations on previous page

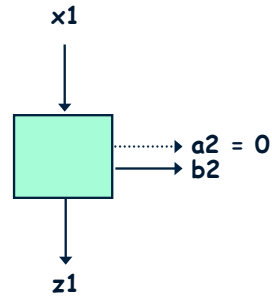


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## Iterative Circuit Synthesis: Mealy-based

Iterative Cell: Gate-Level Implementation

Optimized leftmost cell (= Cell #1): simplify earlier equations



Special condition for cell #1:  $a1 \ b1 = 00!$

$$a2 = b1 = 0$$

$$b2 = a1' x1' + x1' b1 + a1' b1 = x1'$$

$$z1 = x1 a1' + x1' a1 = x1$$

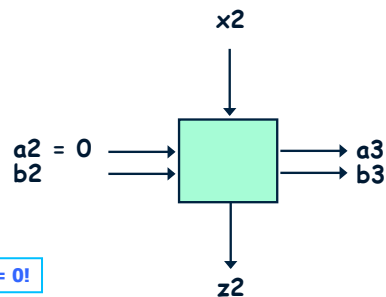
Note: can eliminate  $a2$  output, and simply have a '0'  $a2$  input to next cell (Cell #2)

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## Iterative Circuit Synthesis: Mealy-based

Iterative Cell: Gate-Level Implementation

Optimized second-to-leftmost cell (= Cell #2): propagate simplification



Special condition for cell #2:  $a2 = 0!$

$$a3 = b2$$

$$b3 = a2' x2' + x2' b2 + a2' b2 = x2' + x2' b2 + b2 = x2' + b2$$

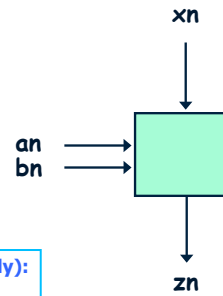
$$z2 = x2 a2' + x2' a2 = x2 \oplus a2 = x2$$

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## Iterative Circuit Synthesis: Mealy-based

Iterative Cell: Gate-Level Implementation

Optimized rightmost cell (= Cell #N): no need for next-state logic -- DELETE!



Special condition for cell #N (Mealy only):  
no next-state logic

$$z_n = x_n a_n' + x_n' a_n = x_n \oplus a_n$$

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